14 Waves

Most of the information we receive is transmitted to us as waves. We rely on waves to bring us Music and TV. Animals move across their environment via wave motion. Eels and snakes employ transverse body waves to push against the water or the ground in order to move. However, earthworms move by using longitudinal waves.

Topic Notes

- *Waves and their Properties*
- *Principle of Supervision of Waves*

WAVES AND THEIR PROPERTIES 1

TOPIC 1

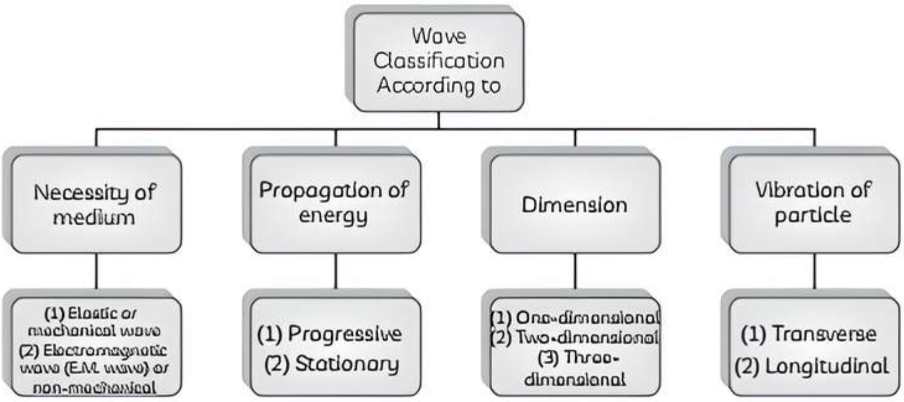
WAVE

A wave is a continuous disturbance, that transfers energy and information, from one point to another without actually moving matter. Water waves are well-known examples. When a pebble is dropped into a pond, it causes the water's surface to ripple. Ripples on the pond's surface move away from where the pebble landed.

Wave Motion

When a particle moves through space, it carries K.E. with itself. Wherever the particle goes, the energy goes with it. (One way of transporting energy from one place to another). There is another way (wave motion) to transport energy from one part of space to another without any bulk motion of material together with it. Sound is transmitted in air in this manner.

Classification of Wave Motion



Based on medium necessity

A wave may or may not require a medium for its propagation. The waves which don't require a medium for their propagation are called non-mechanical *e.g.* light, heat infrared, radio waves etc. On the other hand, the waves which require a medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role in this. Therefore, they are also called elastic waves. For example; water waves, sound waves and seismic waves.

Based on energy propagation

Waves can be divided into two parts on the basis of energy propagation:

(1) Progressive wave

(2) Stationary wave

The Progressive wave propagates with constant velocity in a medium. In stationary waves, particles of the medium vibrate with different amplitudes but energy doesn't propagate.

Related Theory

➥ *If we generate waves in a medium continuously, the particles of the medium oscillate continuously. In this situation, the disturbance produced in the medium is called a progressive wave.*

Based on direction of propagation

Waves can be one, two or three-dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two-dimensional while sound or light waves from a point source are three-dimensional.

Based on motion of particles of medium

Waves are of two types, on the basis of motion of particles of medium:

(1) Longitudinal waves

(2) Transverse waves

In the transverse waves, the direction associated with the disturbance (*i.e.*, motion of the particles of the medium) is at a right angle to the direction of propagation of wave while in the longitudinal wave, the direction of disturbance is along the direction of propagation.

Important

➥ *A wave motion is a kind of disturbance which travels through a medium due to repeated periodic motion of the particles of the medium about their mean position, the motion being transferred continuously from particle to particle.*

Example 1.1: Why longitudinal waves are called pressure waves?

Ans. Because propagation of longitudinal waves through a medium involves changes in pressure and volume of air when compressions and rarefactions are formed.

Example 1.2: Explain, why (or how):

(A) In a sound wave, a displacement node is a pressure antinode and *vice versa*.

(B) Bats can ascertain distances, directions, nature and sizes of the obstacles without any "eyes".

(C) A violin node and a sitar node may have the same frequency, yet we can distinguish between the two nodes.

(D) Solids can support both longitudinal and transverse waves but only longitudinal waves can propagate in gases. [NCERT]

Ans. (A) A node is a point where the amplitude of vibration is the minimum and pressure is the maximum. On the other hand, an antinode is a point where the amplitude of vibration is the maximum and pressure is the minimum. Therefore, a displacement node is nothing but a pressure antinode and *vice versa*.

(B) Bats emit very high-frequency ultrasonic sound waves. These waves get reflected back toward them by obstacles. A bat receives a reflected wave (frequency) and estimates the distance, direction, nature and size of an obstacle with the help of its brain senses.

(C) The overtones produced by a sitar and a violin and the strengths of these overtones are different. Hence, one can distinguish between the nodes produced by a sitar and a violin even if they have the same frequency of vibration.

(D) Solids have a shear modulus. They can sustain shearing stress. Since fluids do not have any definite shape, they yield to shearing stress. The propagation of a transverse wave is such that it produces shearing stress in a medium. The propagation of such a wave is possible only in solids and not in gases.

Both solids and fluids have their respective bulk moduli. They can sustain compressive stress. Hence, longitudinal waves can propagate through solids and fluids.

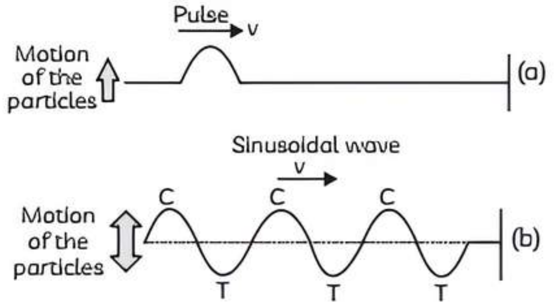
TOPIC 2

TRANSVERSE AND LONGITUDINAL WAVES

Transverse Waves

These are waves in which the individual particles of the medium oscillate perpendicular to the wave's propagation direction.

Consider a horizontal string with one end attached to a rigid support and the other end in your hand fig (a). When we jerk its free end upward, an upward pulse is created that travels along the string towards the fixed end. Each string segment as shown in fig. (b) successively experiences a disturbance about its mean position. If we keep giving up and down jerks. A series of sinusoidal waves begin to travel along the string from the free end.



Each part of the string vibrates up and down while the wave travels along the string. So, the waves in the string are transverse in nature.

A crest is a portion of the medium, which is raised temporarily above the normal position of the rest of particles of the medium when a transverse wave passes.

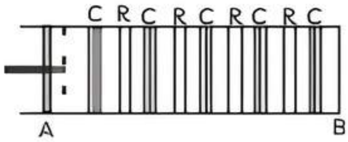
A trough is a portion of the medium, which is depressed temporarily below the normal position of the rest of particles of the medium when a transverse wave passes.

One crest and one trough together form one wave.

Longitudinal Waves

These are the waves in which, the medium's individual particles oscillate in the direction of wave propagation.

Consider a long hollow cylinder AB with one end closed and a movable piston at the other end. When we move the piston quickly to the right, a small layer of air near the piston head is compressed and once compressed, this layer moves to the right and compresses the next layer and the compression soon reaches the other end. If the piston is suddenly moved to the left, the layer adjacent to it is rarefied, resulting in a pressure drop. In the restored pressure, air from the next layer moves. As a result, the next layer is rarefied. In this manner, a pulse of rarefaction moves towards the right.



When we continuously push and pull the piston in a simple harmonic manner, a sinusoidal sound wave travels along the cylinder in the form of alternate compressions and rarefactions, denoted by the letters C, R, C, R and so on. The wave is a longitudinal wave because the oscillations of an element of air are parallel to the direction of wave propagation. As a result, sound waves generated in air are longitudinal waves.

Example 1.3: **A transverse harmonic wave on a string is described by,**

$$y(x, t) = 3\sin\left(36t + 0.018x + \frac{\pi}{4}\right),$$

where x and y are in cm and t in sec. The positive direction of x-axis is from left to right.

(A) Is this a travelling or stationary wave? If it is travelling, what are the speed and direction of its propagation?

(B) What are its amplitude and frequency?

(C) What is the initial phase at the origin?

(D) What is the least distance between two successive crests in the wave? [NCERT]

Ans. Given, the wave equation,

$$y(x, t) = 3\sin\left(36t + 0.018x + \frac{\pi}{4}\right)$$

The general equation of a plane progressive wave is,

$$y(x, t) = a\sin\left[\frac{2\pi}{4}(vt + x) + \phi\right]$$

Comparing, we get,

(A) The given wave is a travelling wave, moving from right to left.

Equating coefficient of t,

$$\text{velocity, } v = \frac{36}{0.018}$$

$$= 2000 \text{ cm/s}$$

(B) Amplitude,

$$A = 3 \text{ cm},$$

$$\frac{2\pi}{4} = 0.018$$

$$\lambda = \text{wavelength}$$

$$\lambda = \frac{2\pi}{0.018}$$

$$= 349 \text{ cm}$$

Frequency,

$$\nu = \frac{v}{\lambda}$$

$$= \frac{2000}{349}$$

$$= 5.73 \text{ Hz}$$

(C) Initial phase, $= \phi = \frac{\pi}{4}$ radian

(D) Least distance between two successive crests

$$= \text{wavelength}$$

$$= 349 \text{ cm}; = 3.5 \text{ m}$$

Example 1.4: **You have learnt that a travelling wave in one dimension is represented by a function $y = f(x, t)$, where x and t must appear in the combination, $x - vt$ or $x + vt$, i.e., $y = f(x \pm vt)$. Is the converse true? Examine, if the following functions for y can possibly represent a travelling wave:**

(A) $(x - vt)^2$

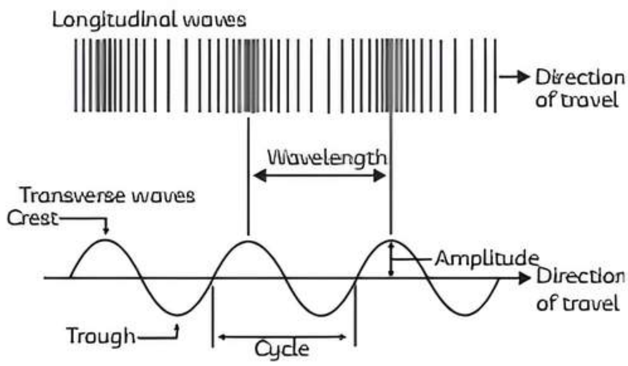
(B) $\log\left[\frac{(x + vt)}{x_0}\right]$

(C) $\frac{1}{(x + vt)}$ **[NCERT]**

Ans. The converse is not true. For any function to represent a travelling wave, it should be finite everywhere and at all times. Only function (C), satisfies this condition, the remaining functions cannot possibly represent a travelling wave.

Example 1.5: **Case Based:**

Transverse waves form when the particles in a medium vibrate at an angle to the direction of wave motion energy propagation, the transverse wave These are passed down as crests and troughs. Longitudinal waves form when the particles of a medium vibrate in the direction of wave motion; this type of wave is known as a longitudinal wave. These are transmitted as compressions and rarefactions, and the wave is also referred to as a pressure or compression wave. Waves on a spring or sound waves in the air are examples of longitudinal waves.



(A) The particles of the medium in a transverse wave:

(a) vibrate in a direction perpendicular to the direction of the propagation

(b) vibrate in a direction parallel to the direction of the propagation

(c) move in circle

(d) move in ellipse

(B) Some examples of wave motion are provided below. In each case, specify whether the wave motion is transverse, longitudinal or a combination of the two:

(i) kink motion produced by displacing one end of a long coil spring sideways.

(ii) vibrating quartz crystal generates ultrasonic waves in the air.

(iii) motorboat sailing in water creates waves.

(iv) waves created in a liquid-filled cylinder by moving its piston back and forth.

(C) Assertion (A): Sound waves cannot propagate through vacuum but light waves can.

Reason (R): Sound waves cannot be polarised but light waves can be polarised.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

(D) A progressive wave with a frequency of 500 Hz is moving at a speed of 360 m/s. How far apart are two points that are 60 degrees out of the phase?

(E) A stone thrown from the top of a 300-metre-tall tower hat splashes into the water of a pond near the tower's base. Given that, the speed of sound in air is 340 ms^{-1}, when is the splash heard at the top? (g = 9.8 milliseconds).

Ans. (A) *(a) vibrate in a direction perpendicular to the direction of the propagation*

Explanation: In a transverse wave, the particles of the medium vibrate in a direction perpendicular to the direction of the propagation. In a Transverse wave, all points on a wave oscillate along paths at right angles to the direction of the wave's propagation. Surface ripples on water and electromagnetic waves like radio and light waves are such examples.

(B) (i) Transverse wave motion, because the vibrations of particles (kinks) of the spring are at right angles to the direction of wave propagation.

(ii) Longitudinal wave motion, because the molecules of the liquid vibrate to and fro about their mean position along the direction of propagation of the wave.

(iii) Combination of longitudinal and transverse waves, because the propeller of the molar boat cuts the water surface laterally and also pushes it in the backward direction.

(iv) Ultrasonic waves produced by a quartz crystal in air are longitudinal because the molecules of air vibrate to and fro about their mean positions along the direction of propagation of wave due to vibrations of quartz crystal.

(C) *(b) Both A and R are true and R is not correct explanation of A.*

Explanation: Sound waves cannot propagate through vacuum because sound waves are mechanical waves. Light waves can propagate through vacuum because light waves are electromagnetic waves. Since sound waves are longitudinal waves, the particles move in the direction of propagation, therefore these waves cannot be polarised.

(D) For a wave,

$$v = f.d$$

so,

$$d = \frac{v}{f} = \frac{360}{500}$$

$$d = 0.72 \text{ m}$$

Phase difference,

$$\Delta\phi = 60° = \left(\frac{\pi}{180}\right) \times 60 = \frac{\pi}{3} \text{ rad}$$

So, path difference,

$$\Delta = \frac{4}{2\pi}(\Delta\phi)$$

$$= \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12 \text{ m}$$

(E) The time at which, the stone hits the ground can be found by,

$$s = ut + \frac{1}{2}gt^2$$

$$\therefore \quad 300 = 0 \times t + \frac{1}{2} \times 9.8 \times t^2$$

$$t = 7.82 \text{ s}$$

Now, the time for sound to travel 300 m is,

$$t = \frac{300}{340}$$

$$= 0.88 \text{ s}$$

Therefore, total time will be addition of the two times.

$$t = 7.82 + 0.88 = 8.7 \text{ s}$$

TOPIC 3

DISPLACEMENT RELATION IN A PROGRESSIVE WAVE

Progressive Waves

"A wave that travels from one point of the medium to another is called a progressive wave". A progressive wave may be transverse or longitudinal.

Displacement Relation in a Progressive Wave

Suppose, a simple harmonic wave starts from the origin O and travels along the positive direction of the x-axis with speed v. Let the time be measured from the instant when the particle at the origin O is passing through the mean position.

Let $y(x, t)$ be the displacement of an element about the y-axis at position x and time t. If the wave is periodic and sinusoidal, the displacement of the element from the y-axis at position x and time t can be given as,

$$y(x, t) = a \sin(kx - \omega t + \phi) \quad \text{...(i)}$$

The term k in the argument of sine function means equivalently that we are considering a linear combination of sine and cosine functions:

$$y(x, t) = A \sin(kx - \omega t) + B \cos(kx - \omega t) \quad \text{...(ii)}$$

From both the above equation,

$$a = \sqrt{A^2 + B^2}$$

and $$\phi = \tan^{-1} \frac{B}{A}$$

The equations (i) and (ii) represent the transverse wave moving along the x-axis, where $y(x, t)$ gives the displacement of the elements of the string at a position x at any time t. Hence, the shape of the wave can be determined at any given time.

$$y(x, t) = a \sin(kx + \omega t + \phi)$$

The equation represents a wave travelling in the negative direction of the x-axis.

Where, $y(x, t)$ = displacement as a function of position x and time t

a = amplitude of a wave

ω = angular frequency of the wave

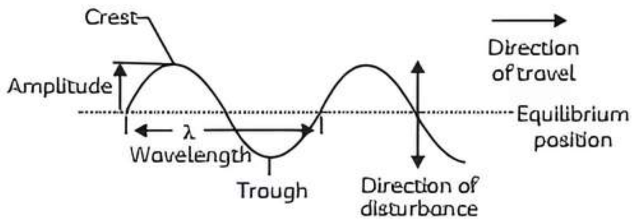
k = angular wave number

$kx - \omega t + \phi$ = initial phase angle ($a + x = 0, t = 0$)

Consider the sinusoidal graph shown. Here, the plot shows a wave travelling in the positive x direction.

Important

➥ *When a progressive wave propagates in a medium, then, at any instant, all the particles of the medium oscillate in the same way but the phase of oscillation changes from particle to particle.*



The point of maximum positive displacement is called a crest and that of maximum negative displacement is called a trough.

Amplitude

The magnitude of a particle's maximum displacement from the equilibrium position in a wave is defined as its amplitude. It is defined as the maximum displacement of an oscillating particle of the medium from the mean position. It is denoted by the symbol A.

Phase

The phase of the function is defined as the argument ($kx - \omega t + \phi$) of the oscillatory term $\sin(kx - \omega t + \phi)$. It describes the wave's state of motion. Points on a wave that rise and fall in the same direction are said to be in phase with each other. Points on a wave that travel in opposite directions, such that one rises while the other falls are said to be anti-phase with each other.

Wavelength and Angular Wave Number

Wavelength

Wavelength (λ) is the distance between two identical points on a wave, such as a crest or a trough, that are parallel to the wave's propagation direction. It is also the distance over which the wave shape repeats itself. It's measured in meters (m).

Angular Wave Number

The wave number expresses a wave's special frequency in terms of cycles per unit distance. It can also be defined as the number of waves present over a given distance, which is similar to the concept of frequency.

Period, Angular Frequency and Frequency

Time period

The time it takes a particle to complete one oscillation about its mean position. It is represented by the symbol T.

Frequency

The number of oscillations made by the particle in one second is defined as its frequency. It is represented by the symbol ν.

$$\nu = \frac{1}{T}$$

Angular frequency

It is defined as the rate of change of phase.

$$\omega = \frac{2\pi}{T} = 2\pi\nu$$

Example 1.6: **A string of mass 2.50 kg is under a tension of 200 N. The length of the stretched string is 20 m. If the transverse jerk is struck at one end of the string, how long does the disturbance take to reach the other end? [NCERT]**

Ans. Given.

$$M = 2.5 \text{ kg.}$$
$$T = 200 \text{ N.}$$
$$l = 20 \text{ m.}$$

Mass/unit length. $m = \frac{M}{l}$

$$= \frac{2.5}{2.0}$$

$$= 0.125 \text{ kg/m}$$

Velocity. $v = \sqrt{\frac{T}{m}}$

$$= \sqrt{\frac{200}{0.125}}$$

$$v = 40 \text{ m/s}$$

Time taken by disturbance to reach the other end.

$$t = \frac{l}{v} = \frac{20}{40}$$

$$t = 0.5 \text{ s}$$

Example 1.7: **The transverse displacement of a string (clamped at its both ends) is given by,**

$$y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right) \cos(120\pi t)$$

where, x and y are in m and t in s. The length of the string is 1.5 m and its mass is 3×10^{-2} kg.

Answer the following:

(A) Does the function represent a travelling or a stationary wave?

(B) Interpret the wave as a superposition of two waves travelling in opposite directions. What is the wavelength, frequency and speed of propagation of each wave?

(C) Determine the tension in the string. [NCERT]

Ans. (A) The equation given.

$$y(x, t) = 0.06 \sin\left(\frac{2\pi}{3}x\right)\cos(120\pi t)$$

This equation has both harmonic functions of x and t separately. Thus it represents a stationary wave.

(B) A stationary wave is formed when a wave.

$$y_1 = a \sin \frac{2\pi}{3}(vt - x)$$

travelling along +ve direction of x and another reflected wave.

$$y_2 = -a \sin \frac{2\pi}{3}(vt - x) \quad \text{...(i)}$$

travelling along –ve direction of x. superpose on each other.

Resultant wave.

$$y = y_1 + y_2$$

$$= 2a \sin \frac{2\pi}{3} \cos \frac{2\pi}{3} vt \quad \text{...(ii)}$$

Comparing equations (i) and (ii).

$$\frac{2\pi v}{3} = 120\pi$$

$$v = 60 \times 3$$

$$= 180 \text{ m/s}$$

Frequency. $\nu = \frac{v}{3} = \frac{180}{3} = 60 \text{ Hz}$

Both waves have the same wavelength. frequency and speed.

(C) Velocity.

$$v = \sqrt{\frac{T}{m}}$$

$$T = v^2 \times m$$

$$= (180)^2 \times \frac{3 \times 10^{-2}}{1.5}$$

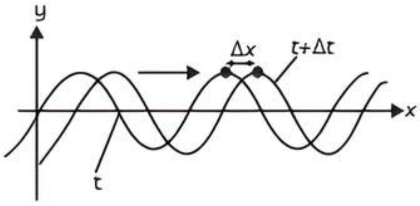
$$= 648 \text{ N}$$

TOPIC 4

THE SPEED OF A TRAVELLING WAVE

Each point of the moving waveform represents a specific phase of the wave and retains its displacement y as the wave moves. As a result an argument of sine must be a constant in the displacement relation.

Consider the motion of the crest of the wave as shown in the figure:



The motion of a fixed phase point on the wave is given by,

$$kx - \omega t = \text{constant}$$

Thus, as time t changes, the position x of the fixed phase point must change. So that the phase remains constant.

Thus, $$kx - \omega t = k(x + \Delta x) - \omega(t + \Delta t)$$

or $$k\,\Delta x - \omega\,\Delta t = 0$$

Taking Δx, Δt vanishingly small, this gives,

$$\frac{dx}{dt} = \frac{\omega}{k} = v$$

Relating ω to T and k to λ,

We get, $$\omega = \frac{2\pi}{T} = 2\pi f$$

$$v = \frac{2\pi v}{2\pi/\omega}$$

$$v = \frac{\omega}{t}$$

$$k = \frac{2\pi}{\lambda}$$

The equation shows the general relation for all progressive waves and shows that in the time required for one full oscillation by any constituent of the medium, the wave pattern travels a distance equal to the wavelength of the wave.

Speed of a Transverse Wave on Stretched String

The speed of any mechanical wave, transverse or longitudinal, is determined by an inertial property of the medium (the ability to store kinetic energy) as well as an elastic property of the medium (to store potential energy).

Consider a transverse pulse generated by a taut string with a linear mass density μ.

Consider a short segment of the pulse Δl, that forms an arc of a circle with radius R. At each end, a force equal to the tension T pulls tangentially on this segment.

Set an observer in the centre of the pulse that moves with the pulse to the right. Any small length dl of the string, as shown, appears to move backwards with a velocity v to the observer.

The string's small mass is now moving in a circular path with radius R and speed v. As a result, the only force acting (ignoring gravity) to provide the required centripetal force is the component of tension along the radius.

The net restoring force on the element is,

$$F = 2T\sin(\Delta\theta)$$

$$\approx T(2\Delta\theta) = T\frac{\Delta l}{R}$$

The mass of the segment is,

$$\Delta m = \mu\Delta l$$

The acceleration of this element towards the centre of the circle is, $a = \frac{v^2}{R}$, where v is the velocity of the pulse.

Using the second law of motion,

$$T\frac{\Delta l}{R} = (\mu\Delta l)\left(\frac{v^2}{R}\right)$$

or $$v = \sqrt{\frac{T}{\mu}}$$

Speed of a Longitudinal Wave (Speed of Sound)

According to Newton's formula,

Velocity of a longitudinal wave in the medium,

$$v_{medium} = \sqrt{\frac{E}{\rho}}$$

Where, E = elastic coefficient of the medium and ρ = density of the medium

(1) **For solid medium:**

$$v_{Solid} = \sqrt{\frac{Y}{\rho}}$$

Here, Y = Young's modulus

(2) **For liquid medium:**

$$v_{liquid} = \sqrt{\frac{B}{\rho}}$$

Here, B = Bulk modulus.

(3) **For gas medium:** The formula for velocity of sound in air was first obtained by Newton. He assumed that sound propagates through air, temperature remains constant. (*i.e.*, the process is isothermal)

So, isothermal elasticity = P

Hence, velocity,

$$v_{air} = \sqrt{\frac{P}{\rho}}$$

Laplace Correction

To eliminate the disparity between theoretical and experimental values of sound velocity, Laplace modified Newton's formula, assuming that sound propagation in air is an adiabatic process.

i.e., Adiabatic elasticity = γP

$$v_{air} = \sqrt{\frac{\gamma P}{\rho}}$$

$$= \sqrt{1.41} \times 279$$

$$= 331.3 \text{ m/s}$$

Which is in good agreement with the experimental value (332 m/s). This in turn establishes that sound propagates adiabatically through gases.

The velocity of sound in air at NTP is 332 m/s which is much lesser than that of light and radiowaves (3×10^8 m/s).

In case of gases,

$$v_{sound} = \sqrt{\frac{E}{\rho}} = \sqrt{\frac{E_v}{\rho}} = \sqrt{\frac{\gamma P}{\rho}}$$

i.e., $$v_{sound} = \sqrt{\frac{\gamma PV}{M}}$$

$$\left[\text{as } m = \frac{\text{mass}}{\text{volume}} = \frac{M}{V}\right]$$

or $$v_{sound} = \sqrt{\frac{\gamma \mu RT}{M}} \quad [\text{as } PV = \mu RT]$$

or $$v_{sound} = \sqrt{\frac{\gamma RT}{M_w}}$$

as $$\mu = \frac{\text{mass}}{M_w} = \frac{M}{M_w}$$

(M_w = molecular weight)

And from kinetic theory of gases,

$$v_{rms} = \sqrt{\frac{3RT}{M_w}}$$

So, $$\frac{v_{sound}}{v_{rms}} = \sqrt{\frac{\gamma}{3}}$$

i.e., $$v_{sound} = \left[\frac{\gamma}{3}\right]^{\frac{1}{2}} v_{rms}$$

i.e., Velocity of sound in a gas is $\left[\frac{\gamma}{3}\right]^{\frac{1}{2}}$ times rms speed of gas molecules (v_{sound}) = v_{rms}.

As velocity of sound is,

$$v_{sound} = \sqrt{\frac{\gamma RT}{M_w}}$$

Velocity of sound in the case of gases at constant temperature depends on the nature of gas, *i.e.*, its atomicity (γ) and molecular weight (M_w).

Important

➥ *The density of a solid is much larger than that of a gas but the elasticity is larger by a greater factor. Hence, longitudinal waves in a solid travel much faster than that in a gas.*

Example 1.8: A stone dropped from the top of a tower of height 300 m high splashes into the water of a pond near the base of the tower. When is the splash heard at the top?

[Speed of sound in air = 340 m/s and g = 9.8 m/s^2]

[NCERT]

Ans. Here, u = 0 m/s

Given, h = 300 m,

g = 9.8 m/s^2,

v = 340 m/s

Let t_1 = time taken by the stone to strike the surface of water.

Using, $$s = ut_1 + \frac{1}{2} g t_1^2$$

$$300 = 0 + \frac{1}{2} \times 9.8 \times t_1^2$$

$$t_1 = \sqrt{\frac{300}{49}}$$

$$= 7.82 \text{ s}$$

And, t_2 = time taken by sound to reach to top of tower,

$$t_2 = \frac{h}{v}$$

$$= \frac{300}{340}$$

$$= 0.88 \text{ s}$$

Total time after which the splash is heard,

$$t = t_1 + t_2$$

$$t = 7.82 + 0.88$$

$$= 8.70 \text{ s}$$

Example 1.9: A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5×10^{-2} kg and its linear mass density is 4×10^{-2} kg/m. What is

(A) The speed of the transverse wave on the string and

(B) The tension in the string? [NCERT]

Ans. (A) Given, $n = 45$ Hz , $M = 3.5 \times 10^{-2}$ kg,

$m = 4 \times 10^{-2}$ kg/m

$$l = \frac{M}{m} = \frac{3.5\times10^{-2}}{4\times10^{-2}} = \frac{7}{8} \text{ m}$$

As it is vibrating in fundamental mode,

$l' = 2l$

$$l = \frac{7}{8},$$

$$l' = \frac{7}{4} = 1.75 \text{ m}$$

Speed, $v = 45 \times 1.75$

$= 78.75$ m/s

(B) $$v = \sqrt{\frac{T}{m}},$$

$T = v^2 \times m$

$= 78.75^2 \times 4 \times 10^{-2} = 248.06$ N

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. **A plane wave is defined by the equation $y = 5\cos\left(\frac{x}{2} - 20t - \frac{\pi}{2}\right)$. The maximum velocity of the medium's particles as a result of this wave is:**

(a) 50 ms^{-1} (b) 100 ms^{-1}
(c) 200 ms^{-1} (d) 175 ms^{-1}

Ans. *(b) 100 ms^{-1}*

Explanation: A = 5 m, ω= 20 rad s^{-1}, Maximum Velocity (v_{max})

= Amplitude × Angular Frequency

$v_{max} = A \times \omega$

$v_{max} = 5 \times 20 = 100 \text{ ms}^{-1}$

2. **A wave train produced by a string has a distance of 25 cm between two consecutive crests. If five full waves pass through any point in one second. The wave's velocity is:**

(a) 125 cm/s (b) 100 cm/s
(c) 0 m/s (d) 50 cm/s

Ans. *(a) 125 cm/s*

Explanation: Wavelength, λ = distance between two consecutive crests = 25 cm.

Frequency, $\nu = 5$.

Hence, $v = \nu \times \lambda$

$v = 5 \times 25$

$= 125$ cm/s

3. **A cohesive string with a mass of 0.1 kg and a length of 3 m hangs from the ceiling. The speed of the transverse wave in the string at the upper end and 0.5 m from the lower end will be:**

(a) 4.413 m/s, 3.16 m/s
(b) 5.477 m/s, 2.18 m/s
(c) 5.477 m/s, 3.16 m/s
(d) 5.477 m/s, 2.18 m/s

Ans. *(c) 5.477 m/s, 3.16 m/s*

Explanation: Velocity of a transverse wave in a stretched string is,

$$v = \sqrt{\frac{T}{m}},$$

where mass per unit length,

$$m = \frac{M}{l} = \frac{0.9}{3}$$

$$= \frac{3}{10}$$

M = weight of part of rope hanging below the point under consideration

$$= \left(\frac{M}{l}\right)xg$$

$$\sqrt{\frac{\left(\frac{M}{l}\right)xg}{m}} = \sqrt{xg}$$

At the upper end,

$$\text{velocity} = \sqrt{3\times10} = 5.477 \text{ m/s}$$

At a point 0.5 m distance from the lower end,

$$\text{velocity} = \sqrt{1\times10} = \sqrt{10}$$

$= 3.16$ m/s

⚠ Caution

➥ *Students must know that in strings, mechanical waves are always transverse when the string is under a tension. In the bulk of gases and liquids mechanical waves are always longitudinal e.g., sound waves in air or water. This is because fluids cannot sustain shear.*

4. **The wave speed in a string is 15 m/s and the frequency is 150 Hz. The phase difference at a distance of 1.5 cm will be:**

(a) $\frac{\pi}{2}$ (b) 0.3π

(c) $\frac{\pi}{4}$ (d) 2π

Ans. *(b) 0.3π*

Explanation: Given that

$$f = 150 \text{ Hz}$$
$$x = 1.5 \text{ cm}$$
$$v = 15 \text{ m/s}$$

Path difference wavelength,

$$\lambda = \frac{v}{f}$$
$$= \frac{15}{150} = 0.1 \text{ m}$$
$$\phi = \frac{2\pi}{\lambda} \times x$$
$$= \frac{2\pi}{0.1} \times 1.5 \times 10^{-2}$$
$$= 0.3\pi$$

5. **A 50 Hz wave travels down a string to its fixed end. When this wave returns after reflection, a node 15 cm away from the fixed end is formed. The speed of the wave (both incident and reflected) is:**

(a) 15 m/s (b) 25 m/s

(c) 20 m/s (d) 5 m/s

Ans. *(a) 15 m/s*

Explanation: Frequency (□) = 50 Hz and distance from fixed end = 15 cm = 0.15 m.

When a stationary wave is produced, the fixed end behaves as a node. Thus, wavelength

$$\lambda = 2 \times 0.15$$
$$= 0.3 \text{ m.}$$

Therefore, velocity v = □λ

$$= 50 \times 0.3$$
$$= 15 \text{ m/s.}$$

Related Theory

➥ *In a liquid the speed lies in between the two i.e., $v_{solid} > v_{liquid} > v_{gas}$.*

6. **During propagation of plane progressive mechanical wave:**

(a) amplitude of all particles is equal

(b) particles of the medium executes SHM

(c) wave velocity depends upon the nature of the medium. non-periodic

(d) all of the above [NCERT Exemplar]

Ans. *(d) all of the above*

Explanation: During propagation of mechanical wave each particle displaces from zero to maximum *i.e.*, upto amplitude. So amplitude of each particle is equal. Verifies the option (b).

For progressive waves, medium particles oscillate about their mean position in which restoring force F ∝ (– y). So motion of medium particles is simple harmonic motion. So verifies the option (c).

For progressive wave, propagating in a medium of density (□) and Bulk modulus k, the velocity (v).

$$v = \sqrt{\frac{k}{\rho}}$$

As the v depends on k and ρ, and k, ρ are different for a different medium, so v of a wave depends on nature of medium, hence, verifies the option (d).

Caution

➥ *Students should know that when a progressive wave propagates in a medium, then, at any instant all the particles of the medium oscillate in the same way but the phase of oscillation changes from particle to particle.*

Related Theory

➥ *Each particle between any two successive crests and troughs is in a different mode of vibration.*

7. **A string of mass 2.5 kg is under a tension of 200 N. The length of the stretched string is 20.0 m. If the transverse jerk is struck at the one end of the string, the disturbance will reach the other end in:**

(a) 1 sec

(b) 0.5 sec

(c) 2 sec

(d) Data given is insuffcient

[NCERT Exemplar]

Ans. *(b) 0.5 sec*

Explanation: M = mass of string = 2.5 kg,

l = 20 m,

= mass per unit length, m

$$= \frac{M}{l} = \frac{2.5}{20}$$
$$= 0.125 \text{ kg/m}$$
$$v = \sqrt{\frac{T}{m}}$$
$$= \sqrt{\frac{200}{0.125}}$$
$$v = \sqrt{1600}$$
$$= 40 \text{ m/s}$$
$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

$$= \frac{20 m}{40 m/s}$$

$$t = \frac{1}{2} \text{ sec}$$

$$= 0.5 \text{ sec}$$

8. Two waves of the same frequency travelling in the same medium in opposite direction when superimposed give rise to:

(a) beats (b) harmonics

(c) standing waves (d) resonance

[Delhi Gov. QB 2022]

Ans. *(c) standing waves*

Explanation: In a medium, two waves move in directions that are mutually incompatible. The observed phenomenon is a stationary wave when it is overlaid.

Assertion – Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below.

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

9. Assertion (A): **The amplitude of vibration in a stationary wave is independent of the particle's position.**

Reason (R): **A particle of medium vibrates in the same phase but with different amplitude between two consecutive nodes in a standing wave pattern.**

Ans. *(c) A is true but R is false.*

Explanation: There are points at regular intervals which are never displaced and are called nodes. And where the particles periodically attain maximum or minimum values are called antinodes. In stationary waves,

(1) All the particles are in a state of vibration except nodes.

(2) All the particles between two consecutive nodes have the same phase but differ in amplitude.

(3) The particles on the opposite side of nodes move in opposite directions. Hence, they are in opposite phases.

In a standing wave, all particles move at different amplitudes. Amplitude of a standing wave is.

$$y = aA \cos kx$$

Amplitude depends on location x.

10. Assertion (A): **When longitudinal waves propagate through a medium, it causes displacement of particles of motion along the direction of wave propagation and is called pressure waves.**

Reason (R): **Propagation of longitudinal waves through a medium involves changes in pressure (density of medium particles) when compressions and rarefactions are formed.**

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: Longitudinal waves are waves in which the displacement of the medium is in the same direction as or the opposite direction to the direction of propagation of the wave. Mechanical longitudinal waves are called as pressure waves because they produce increases and decrease the pressure. When longitudinal waves propagate through a medium, it causes displacement of particles of motion along the direction of wave propagation. Hence, at some points, particles are close to each other while at some points, they are farther apart. This creates a density difference in medium, due to which a pressure difference arises in the material medium. Since, propagation of longitudinal waves through a medium creates pressure disturbances in the medium. Hence, these waves are called pressure waves.

11. Assertion (A): **The particle velocity in a transverse wave is perpendicular to the direction of wave velocity.**

Reason (R): **Energy is always transferred in the direction of wave propagation in a wave motion.**

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: In a transverse wave, particles oscillate perpendicular to the axis of propagation of the wave. The particles move up and down and wave moves perpendicular to its direction. So we can say, the direction of velocity is perpendicular to the direction of propagation of waves.

In any vibration there exists Kinetic Energy and Potential Energy. As the vibration gets transferred, the energy also gets transferred.

12. Assertion (A): **Compression and rarefaction involve changes in density and pressure.**

Reason (R): When particles are compressed, density of the medium increases and when they are rarefied, density of the medium decreases.

[Delhi Gov. QB 2022]

Ans. (a) *Both A and R are true and R is the correct explanation of A.*

Explanation: Particles move closer to one another in an area of the medium known as compression, where their average distance from one another decreases. As a result, the volume temporarily decreases and the medium's density rises as a result.

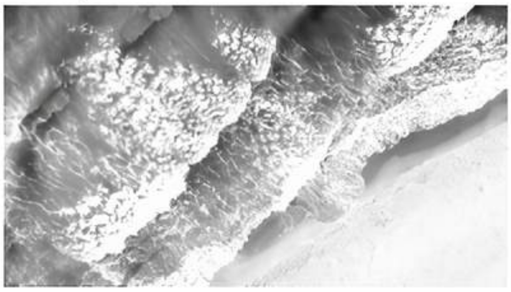
Similar to rarefaction, which results in a drop in density when particles are spaced further apart.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

13. *Water waves near the shore give the appearance of being transverse waves. Actually, are more closely by a combination of transverse and longitudinal waves. This becomes apparent when one watches a small Floating in the water-it not only bobs up and down but also move to-and-fro. The frequencies and wavelengths of these two simultaneous, but mutually perpendicular, waves are the same but their amplitudes, of course, need not be the same.*



(A) If the observed to crest distance of the travelling water waves is 3.0 m and the speed of the waves toward the shore is 2.0 m/s, The frequency of the waves will be:

(a) 0.667 Hz (b) 0.867 Hz
(c) 0.650 Hz (d) 0.600 Hz

(B) The equation of spherical progressive wave is:

(a) $y = A \sin (kx - \omega t)$

(b) $y = \frac{A}{r} \sin (kx - \omega t)$

(c) $y = \frac{A}{\sqrt{r}} \sin (kx - \omega t)$

(d) $y = \frac{A}{2r} \sin (kx - \omega t)$

(C) If the speed of longitudinal mechanical waves in water is 1400 m/s, then the Bulk modulus of elasticity of water is. (Density of water 1 g/cm^3)

(a) 1.96×10^9 N/m^2
(b) 2.08×10^9 N/m^2
(c) 5.64×10^9 N/m^2
(d) 1.02×10^9 N/m^2

(D) The velocity of a pulse in a rope of mass/length, μ = 3.0 kg/m and the tension is 25 N is:

(a) 2.00 m/s (b) 2.59 m/s
(c) 2.89 m/s (d) 3.12 m/s

(E) A transverse wave in a cord of length is L = 3.0 m and mass M = 12.0 g travelling at 6000 cm/s. The tension in the cord is:

(a) 14.4 N (b) 15.2 N
(c) 15.8 N (d) 16.3 N

Ans. (A) *(a) 0.667 Hz*

Explanation: The distance of the travelling water waves, $\lambda = 3.0$ m

And the speed of waves $v = 2.0$ m/s

As we know that,

$$\text{frequency, } f = \frac{v}{\lambda}$$

The frequency of the wave $f = 0.667$ Hz

(B) *(b)* $y = \frac{A}{r} \sin (kx - \omega t)$

Explanation: $I = \frac{P}{A} = \frac{P}{4\pi r^2}$

Amplitude is given as,

$$A = \sqrt{I} = \sqrt{\frac{P}{4\pi r^2}}$$

$$A \propto \frac{1}{r}.$$

(C) *(a)* 1.96×10^9 *N/m^2*

Explanation: Speed of longitudinal wave is,

$$v = \sqrt{\frac{B}{\rho}}$$

$$140000 \text{ cm/s} = \sqrt{\frac{B}{1}}$$

$$\Rightarrow \quad B = 1.96 \times 10^{10} \text{ dyne/cm}^2$$
$$= 1.96 \times 10^9 \text{ N/m}^2$$

(D) *(c) 2.89 m/s*

Explanation: For transverse waves in a cord, the velocity of propagation is given by.

$$v_P = (S)\ \frac{1}{2}$$

$$= [25\ N \times 3.0\ kg/m]\ \frac{1}{2}$$

$$= 2.89\ m/s$$

(E) *(a) 14.4 N*

Explanation: For transverse waves in a cord, the velocity of propagation is given by.

$$v_P = (S)\frac{1}{2}$$

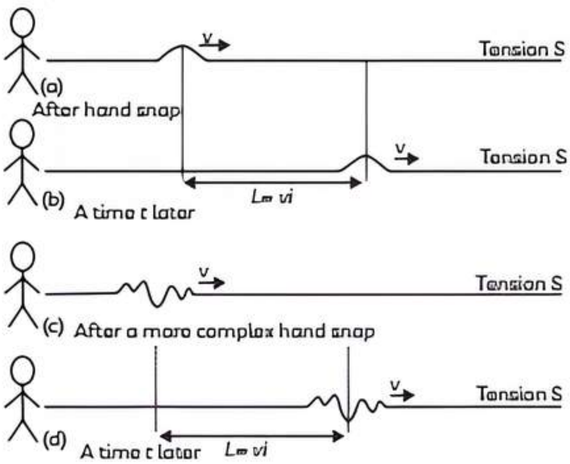
So, by the equation,

$$S = \mu v_{p1} = MLv_{p1}$$

$$= [0.012\ kg \times 3.0\ m]\ 60\ m/s)^2$$

$$= 14.4\ N$$

14. *Consider a student holding one end of a very long cord under tension S, with the far end attached to a wall. If the student suddenly snaps her hand upward and back down, while keeping cord under tension, a pulse, something like that shown in figure (a) will appear to rapidly travel along the cord away from the student. If the amplitude of the pulse (its maximum vertical displacement) is not large compared to its length, the pulse will travel at a constant speed, v, until it reaches the tied end of the cord. In general, shape of the pulse remains the same as it travels in figure (b)], and its size diminishes only slightly (due to thermal losses) as it propagates along the cord. By rapidly shaking her hand in different ways, the student can have pulses of different shapes [e.g., in figure (c)] travelling down the cord. As long as the tension S, in the cord is the same for each such snap, and the amplitudes are not large, the speed of all the pulses in the cord will be the same no matter what their shapes.*



(A) **For the cases of the figure, in what direction are the molecules of the cord moving as they pass by?**

(B) **If actual molecules of cord are not travelling the pulse, what is the K.E.?**

(C) **What qualitative explanation can you give for this phenomenon?**

Ans. (A) We can understand the motion of the cord molecules as the pulse approaches a point in the cord and passes by. First, the molecules at a given horizontal point on the cord move upward, until the maximum of the pulse passes the point, at which the molecules are at the maximum vertical displacement (the amplitude), then the molecules move back down until they return to their normal position as the pulse passes by. Thus, the molecules move perpendicular to the direction in which the pulse moves.

(B) The shape of the pulse travels as one set of molecules after another goes through the vertical motion described in part (A). The pulse carries energy, the vertical kinetic energy of the moving molecules and the associated potential energy due to the momentary stretching of the cord, in the pulse region.

(C) As the tension in the cord, is increased forces between adjacent molecules get stronger, resisting the effort to pull the cord apart. When the student snaps the end of the cord upward the adjacent molecules are forced upward as well and so are the next of molecules and so on. All the molecules in the cord don't move upward at the same instant, however, it takes some time for each succeeding set of molecules to feel the resultant force caused by the slight motion of the prior set away from them. While the successive groups of molecules are being pulled upward, the student snaps her hand back down, so the earlier molecules are reversing direction and moving back down. The net effect is that successive sets of molecules down the length of the cord start moving upward while further back other sets are feeling the pull back down. This process the pulse to, in effect, reproduces itself over and over again down the cord.

VERY SHORT ANSWER Type Questions (VSA)

[1 mark]

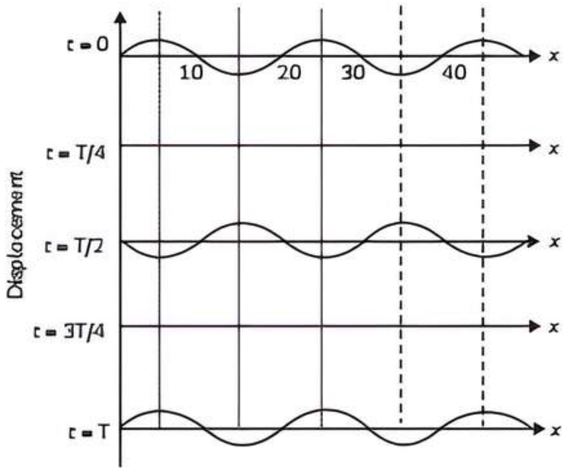
15. As the ocean waves are transverse waves travelling in concentric circles of the ever-increasing radius. When they hit the shore, they always remain at their initial state. Elucidate the reason.

Ans. Waves produced on the surface of water are transverse in nature. When such waves are produced in water they spread out. Till the ocean waves reach the beach shore, they acquire such a large radius of curvature that they may be assumed as plane waves. Hence, ocean waves hit the beach normally to the shore.

16. It's been observed that lots of explosions usually occur in the universe but none of its sounds reaches the earth and we never came to know about those explosions. Elucidate the reason behind this.

Ans. This is because no material medium is present over a long distance between the earth and planets and in the absence of a material medium for propagation, sound waves cannot travel. Sound needs a material medium for its propagation from one place to another place. In other words, sound cannot travel through a vacuum. Since, there is a region between the planets and the earth, where there is a vacuum, the sound of explosions taking place on other planets cannot pass through this vacuum.

17. The wave pattern on the stretched string is shown in the figure. Interpret what kind of wave this is and find its wavelength.



[NCERT Exemplar]

Ans. The displacement of medium particles at distance 10, 20, 30, 40 and 50 cm are always rest which is the property of nodes in a stationary wave. At $t = \frac{T}{4}$ and $\frac{3T}{4}$, all particles are at rest which is in stationary wave when the particle crosses its mean position.

So, the graph of waves shows stationary waves. The wave at $x = 10, 20, 30, 40$ cm there are nodes and distance between successive nodes is $\frac{T}{2}$.

$$\frac{T}{2} = (30-20) \text{ or } = 10 \text{ cm}$$

18. When two waves of almost equal frequencies n_1 and n_2 reached at a point simultaneously. What will be the time interval between successive maxima? [NCERT Exemplar]

Ans. Here frequencies of vibrations are nearly equal but exactly different $n_1 \equiv n_2$. So, beats are formed in the medium when they produce sound, the number of let $n_2 > n_1$.

Then number of beats per second i.e., frequency of maxima,

$$n = n_2 - n_1$$

So, time period of maxima or beats,

$$= \frac{1}{n} = \frac{1}{n_2 - n_1} \text{ seconds}$$

19. The velocity of sound in a tube containing air at 27°C and pressure of 76 cm of Hg is 330 ms^{-1}. What will be its velocity, when pressure is increased to 152 cm of mercury and temperature is kept constant?

[Delhi Gov. QB 2022]

Ans. The velocity (v) of sound in air is proportional to the square root of its pressure. If the temperature is kept constant then we need not bother about it.

$$\frac{v_2}{v_1} = \sqrt{\frac{P_2}{P_1}}$$

$$v_2 = v_1 \times \sqrt{\frac{P_2}{P_1}}$$

$$= 330 \times \sqrt{\frac{100}{76}}$$

$$= 378.53$$

The velocity will be 378.53 m/s.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

20. **A wave propagates down a long rope, which hangs freely from a support. As it propagates, what happens to the speed of the wave?**

Ans. A wave can be thought of as a disturbance or oscillation that travels through space-time, accompanied by a transfer of energy. The direction a wave propagates is perpendicular to the direction it oscillates for transverse waves. A wave does not move mass in the direction of propagation; it transfers energy. The speed of a pulse or wave on a string under tension can be given as:

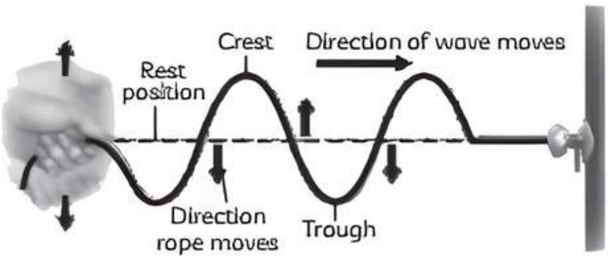
$$v = \sqrt{\frac{T}{\mu}}$$

Where, μ is mass per unit length of rope.
At a distance x from the lower end, we will find tension which will be, $T = \mu g x$
So, equation will become

$$v = \sqrt{\frac{\mu g x}{\mu}} = \sqrt{gx}$$

21. **If you twirl one end of a horizontally stretched rope in a circle in a plane perpendicular to the rope, a wave will propagate along the rope. Is this wave transverse, longitudinal or some combination of the two or none of these?**

Ans. A transverse wave is a wave, in which the disturbance is perpendicular to the direction the wave travels. For example, a wave travelling through a rope is a transverse wave. By shaking at one end of rope up and down. The dotted line shows, where the rope was before it was shaken. This is the rest position. When you shake the rope, the particles in the rope move up and down and the wave moves forward or away from the source of energy. The rope moves in a direction that is perpendicular or at right angles to the direction in the wave moves. All transverse waves move like this.



The highest point on a transverse wave are crests and the lowest point are troughs. As the transverse wave moves through a rope, it makes crests and troughs in the rope.

22. **The equation of plane progressive wave is given by, $y = 0.6 \sin 2\pi\left[t + \frac{x}{2}\right]$. On reflection from a denser medium, its amplitude becomes $\frac{2}{3}$rd of the amplitude of the incident wave. What is the equation of reflected wave? [NCERT Exemplar]**

Ans. After reflection of wave changes by phase 180°.

$$y_i = 0.6 \sin 2\pi\left[t + \frac{x}{2}\right]$$

$$y_r = \left(\frac{2}{3} \times 0.6\right) \sin 2\pi\left[\pi + t + \frac{x}{2}\right]$$

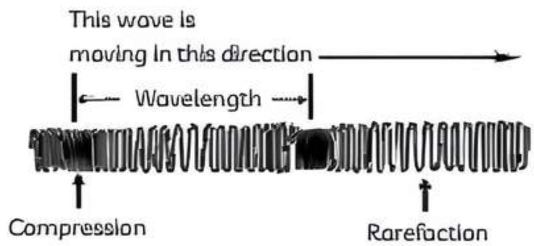
$$y_r = -0.4 \sin 2\pi\left[t + \frac{x}{2}\right]$$

23. **If an explosion takes place at the bottom of a lake or sea, will the shock waves in water be longitudinal or transverse?**

[Delhi Gov. QB 2022]

Ans. Longitudinal waves are created in the water when an explosion occurs at the lake's bottom.

Longitudinal waves are waves in which the displacement of the medium is either parallel to or perpendicular to the wave's direction of propagation. Because mechanical longitudinal waves create compression and rarefaction while passing through a material, they are also known as compression waves. The displacement of the medium in longitudinal waves is parallel to the wave's propagation. The diagrammatic depiction is shown below.



SHORT ANSWER Type-II Questions (SA-II)

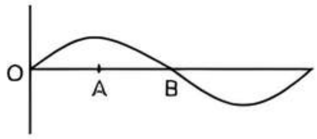
[3 marks]

24. At the two ends of a stretched string, two wave vibrations identical in shape but inverted with respect to each other are produced. When the pulse reaches the centre, the string becomes completely straight. How does the energy released during the generation of two pulses affect the string?

Ans. When the pulses reach the middle, though the string becomes completely straight, the particles of the string there do not stop but have speed. Due to the speed, the pulses again emerge out after a few moments. So the energy of the two pulses at the middle is in the form of kinetic energy of the particles.

25. During the propagation of plane on which factors does the progressive mechanical wave depends? [NCERT Exemplar]

Ans. During propagation of mechanical wave, each particle displaces from zero to maximum *i.e.*, upto amplitude. So the amplitude of each particle is equal.



Each particle between any two successive crests and troughs is in different modes of vibration *i.e.*, in different phase rejects. For progressive wave, medium particles oscillate about their mean position in which restoring force, $F \propto (-y)$.

So, motion of medium particles is simple harmonic motion.

For progressive wave propagating in a medium of density (ρ) and Bulk modulus (k), the velocity (v),

$$v = \sqrt{\frac{k}{\rho}}$$

As the v depends on k and ρ, and k, ρ are different for a different medium. So, velocity of wave depends on nature of the medium.

26. The displacement of elastic wave is given by the function, $y = 3 \sin \omega t + 4 \cos \omega t$, where y is in cm and t is in second, calculate the resultant amplitude. [NCERT Exemplar]

Ans. Given $y = 3 \sin \omega t + 4 \cos \omega t$ ...(i)

Let, $3 = a \cos \phi$ ...(ii)

and $4 = a \sin \phi$ ...(iii)

Then, $y = a \cos \phi \sin \omega t + a \sin \phi \cos \omega t)$

$y = a \sin (\omega t + \phi)$

From eqn. (ii) and (iii),

$$\tan \phi = \frac{4}{3}$$

or $$\phi = \tan^{-1} \frac{4}{3}$$

On squaring and adding (ii) and (iii) equations,

$$a^2 \cos^2 \phi + a^2 \sin^2 \phi = 3^2 + 4^2$$

$$a^2 = 9 + 16$$

$$a^2 = 25$$

or $a = 5$

$y' = 5 \sin (\omega t + \phi)$

where, $$\phi = \tan^{-1} \frac{4}{3}$$

Hence, new amplitude is 5 cm.

27. What do you understand by phase of a wave? How does the phase change with time and position? [Delhi Gov. QB 2022]

Ans: A wave's location at a specific instant on a waveform cycle is known as its phase. Using either degree (0–360) or radians (0–2), it offers a measurement of the exact location of the wave within its cycle. A phase's radian is approximately 57.3 degrees.

Let if at time T_1, the wave's location along the vertical line was, and at time T_2, it was, then the wave's position along the vertical line moved by 1/4 wavelength, or 90 degrees, rather than the wavelength from T_1 to T_2. "Phase shift" is the name given to this alteration.

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

28. A transverse harmonic wave on a string is described by,

$$y(x, t) = 3.0 \sin \left(36t + 0.018\, x + \frac{\pi}{4}\right)$$

where, x and y are in cm and t in s. The positive direction of x is from left to right.

(A) Is this a travelling wave or a stationary wave? If it is travelling, what are the speed and direction of its propagation?

(B) **What are its amplitude and frequency?**

(C) **What is the initial phase at the origin?**

(D) **What is the least distance between two successive crests in the wave?**

Ans. The equation of the form,

$$y(x, t) = A \sin\left(\frac{2\pi}{4}(vt + x) + \phi\right) \quad \text{...(i)}$$

represents a harmonic wave of amplitude A, wavelength l and travelling from right to left with a velocity v. Now, the given equation for the transverse harmonic wave is,

$$y(x, t) = 3.0 \sin\left(\frac{36}{0.018}(t + x) + \frac{\pi}{4}\right) \quad \text{...(ii)}$$

(A) Since the equation (i) and (ii) are of the same form, the given equation also represents a travelling wave propagating from right to left. Further, the coefficient of t gives the speed of the wave. Therefore,

$$v = 2000 \text{ cms}^{-1}$$
$$= 20 \text{ ms}^{-1}.$$

(B) Obviously, amplitude,

$$A = 3.0 \text{ cm}$$

Further,

$$\frac{2\pi}{4} = 0.018$$

or $$= \frac{2\pi}{0.018} \text{ cm}$$

$$v = \frac{v}{2\pi} = \frac{2000}{2\pi} \times 0.018$$

$$= 5.73 \text{ s}^{-1}$$

(C) Initial phase at the origin,

$$\phi = \frac{\pi}{4} \text{ rad}$$

(D) Least distance between two successive crests in the wave is equal to wavelength.

Therefore, $$x = \frac{2\pi}{0.018}$$

$$= 349.0 \text{ cm} = 3.49 \text{ m}$$

29. **In a given progressive wave, $y = 5 \sin(100 \pi t - 0.4 \pi x)$, where x and y are in m and t in seconds, what is the:**

(A) **Amplitude**

(B) **Wavelength**

(C) **Frequency**

(D) **Wave velocity**

(E) **Particle velocity amplitude?**

[NCERT Exemplar]

Ans. Standard form of the progressive wave travelling in + x-direction (kx and ωt have opposite sign is given) eqn. is,

$$y = a \sin(\omega t - kx + \phi)$$
$$y = 5 \sin(100\pi t - 0.4\pi x).$$

(A) Amplitude, $a = 5$ m

(B) Wavelength λ,

$$k = \frac{2\pi}{\lambda}$$
$$k = 0.4\pi$$
$$= \frac{2\pi}{k} = \frac{2 \times \pi}{0.4\pi} = 5 \text{ m}$$

(C) Frequency,

$$\omega = 2\pi v,$$
$$v = \frac{\omega}{2\pi}.$$
$$\omega = 100\pi$$
$$v = \frac{100\pi}{2\pi} = 50 \text{ Hz}$$

(D) Wave velocity,

$$v = 50 \times 5$$
$$= 250 \text{ m/s}$$

(E) Particle (medium) velocity in the direction of amplitude at a distance x from source.

$$y = 5 \sin(100\pi t - 0.4\pi x).$$
$$\frac{dy}{dt} = 5 \times 100\pi \cos(100\pi t - 0.4\pi x)$$

For maximum velocity of the particle is at its mean position.

$$\cos(100\pi t - 0.4\pi x) = 1$$
$$100\pi t - 0.4\pi x = 0$$
$$\left(\frac{dy}{dt}\right)_{max} = 5 \times 100\pi \times 1$$

v_{max} of medium particle $= 500\pi$ m/s

30. **Consider a steel cable of diameter, D = 2.0 mm and under a tension of, S = 15 kN. (For steel, Y = 1.96×10^{11} Pa, $\rho = 7860$ kg/m^3).**

(A) **Find the speed of transverse waves in the cable.**

(B) **Compare the answer to part (A) with the speed of sound in the cable.**

Ans. (A) We need the mass/length, $\mu = \rho A$,

where A is the cross-sectional area of the cable. $$A = \frac{\pi D^2}{4}.$$

Substituting values in equation we get,

$$\mu = (7860) \times (3.14) \times \frac{(0.0020)}{4}$$
$$= 0.0247 \text{ Kg/m}$$

Substituting into for transverse waves in a cord.

$$v_p = \left(\frac{S}{\mu}\right)^{\frac{1}{2}}$$

$$v_p = \left(\frac{15\times10^3}{0.0247}\right)^{\frac{1}{2}}$$

$$= 779 \text{ m/s}$$

(B) The speed of longitudinal (sound) waves is given for longitudinal waves in a cord is given by,

$$v_p = \left(\frac{Y}{\rho}\right)^{\frac{1}{2}}$$

$$v_p = \left(\frac{1.96\times10^{11}}{7860}\right)^{\frac{1}{2}}$$

$$= 4990 \text{ m/s}$$

which is 6.41 times as fast as the transverse wave.

31. If the pitch of the sound of a source appears to drop by 10% to a moving person, then determine the velocity of motion of the person. Velocity of sound = 30 ms^{-1}.

[Delhi Gov. QB 2022]

Ans. The pitch of the sound of a source appears to drop by 10% to a moving person.

$$\therefore \quad \bar{n} = 0.9n$$

According to Doppler effect, the pitch of a sound that appears to a moving person is given by:

$$\bar{n} = n\frac{v-v_0}{v-v_s}$$

Here, v_s = velocity of the sound source is 0 and $v = 330$ m/s and $\frac{\bar{n}}{n} = \frac{90}{100}$

Using the given formula, we can find the velocity of the person, $v_0 = 33$ m/s.

Apparent frequency

$$v' = \left(\frac{v-v_0}{v}\right)v$$

$$\frac{v'}{v} = \frac{v-v_0}{v}$$

$$\frac{v'}{v} = \frac{900}{1000} = \frac{9}{10}.$$

$$v = 330 \text{ ms}^{-1}$$

$$\therefore \frac{9}{10} = \frac{330-v_0}{330}$$

$$330 - v_0 = \frac{9}{10}\times330 = 297$$

$$v_0 = 330 - 297 = 33 \text{ m/s}$$

NUMERICAL Type Questions

32. A 5.50 kg string is under a tension of 500 N. The stretched string measures 15.0 m in length. How long does it take for the transverse jerk to reach the other end of the string if it is struck at one end? (2m)

Ans.

$$M = 5.50 \text{ kg}$$

$$T = 500 \text{ N}$$

$$l = 15.0 \text{ m}$$

Mass per unit length,

$$\mu = \frac{M}{l} = \frac{5.50}{15}$$

$$= 0.366 \text{ kg/m}$$

The velocity (v) of the transverse wave in the string is given by the relation:

$$v = \sqrt{\frac{T}{\mu}}$$

$$= \sqrt{\frac{500}{0.366}}$$

$$= \sqrt{1366.12}$$

$$= 36.96 \text{ m/s}$$

Time taken by the disturbance to reach the other end,

$$t = \frac{l}{v}$$

$$= \frac{15}{36.96}$$

$$= 0.40 \text{ s}$$

33. The equation of wave,

$$y(x, t) = 0.15 \sin \left(\frac{\pi}{2}(10x - 40t) - \frac{\pi}{4}\right)$$

Find wavelength, the frequency and the wave velocity. (3m)

Ans. The equation may be rewritten as

$$y(x, t) = 0.15 \sin \left(10\pi x - 20\pi t - \frac{\pi}{4}\right)$$

Comparing this with equation of plane progressive harmonic wave.

$$y(x, t) = A \sin (kx - \omega t + \phi)$$

wave number,

$$k = \frac{2\pi}{4}$$

$$= \frac{10\pi \text{ rad}}{m}.$$

$$\lambda = 0.2 \text{ m}$$

The angular frequency is,

$$\omega = 2\pi f$$

$$= \frac{20\pi \text{ rad}}{s}.$$

$$f = 10 \text{ Hz}$$

The wave velocity is,

$$v = \frac{\omega}{k} = \frac{20\pi}{10\pi} = 2$$

$= 2 \text{ ms}^{-1}$ in the positive direction.

PROPERTIES OF WAVES 2

TOPIC 1

THE PRINCIPLE OF SUPERPOSITION OF WAVES

When two or more waves pass through the same medium, the displacement of any medium element is the algebraic sum of the displacements caused by each wave; this is known as the principle of wave superposition.

$$y = \sum_{i=1}^{n} f_i(x - vt)$$

Assuming a wave travelling along a stretched string is

$$y_1(x, t) = a \sin(kx - \omega t)$$

and another wave shifted from the first by phase ϕ is

$$y_2(x, t) = a \sin(kx - \omega t + \phi).$$

Applying superposition principle, the resultant wave will be the algebraic sum of the two waves.

$$y = y_1 + y_2.$$

$$y(x, t) = a \sin(kx - \omega t) + a \sin(kx - \omega t + \phi)$$

On solving we get,

$$y(x, t) = 2a \cos \frac{\phi}{2} \sin(kx - \omega t + \frac{\phi}{2})$$

For constructive interference

$$\phi = 0, 2\pi, 4\pi, \ldots$$

For destructive interference,

$$\phi = \pi, 3\pi, 5\pi, \ldots$$

Principle of superposition holds for all types of waves, *i.e.*, mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

Due to superposition of waves, the following phenomenon can be seen:

(1) **Interference:** Superposition of two waves having an equal frequency and nearly equal amplitude.

(2) **Beats:** Superposition of two waves of nearly equal frequency in the same direction.

(3) **Stationary waves:** Superposition of equal waves from opposite direction.

(4) **Lissajous' figure:** Superposition of perpendicular waves.

Important

➥ *Superposition phenomena applicable for all vector quantities.*

➥ *For the resultant wave in interference phenomenon frequency, wavelength and velocity is identical to initial or super-posing waves but their amplitude and initial phase is changed.*

Caution

➥ *Students should know that superposition phenomena are applicable for all types of waves.*

Example 2.1: **Two sitar strings A and B playing the note 'Ga' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string A is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of A is 324 Hz, what is the frequency of B? [NCERT]**

Ans. Let original frequencies of A and B be v_A and v_B respectively.

$$v_A - v_B = \pm 6$$

$$v_B = v_A \pm 6$$

$$= 324 \pm 6$$

$$= 330 \text{ or } 318 \text{ Hz}$$

When the tension is reduced, frequency is also reduced as, $v \propto \sqrt{T}$

As the no. of beats decreases

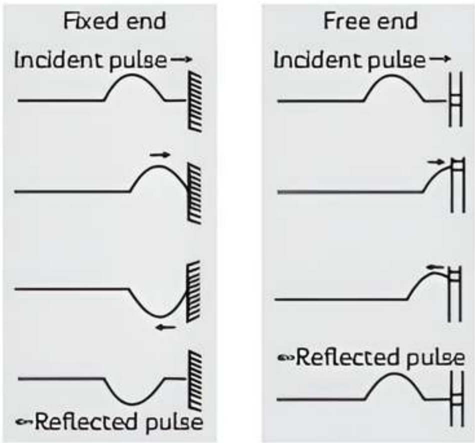
$$v_B = 324 - 6$$

$$= 318 \text{ Hz}$$

TOPIC 2

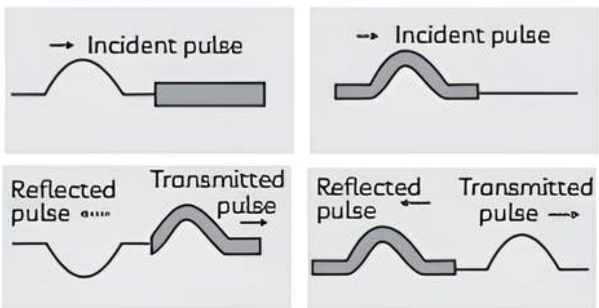
REFLECTION OF WAVES

Whenever a travelling wave reaches a boundary, part or all of the wave will be reflected. For example, consider a pulse travelling on a string fixed at one end (figure). When the pulse reaches the fixed wall, it will be reflected. Since, the support attaching the string to the wall is assumed to be rigid, it does not transmit any part of the disturbance to the wall. Note that, the reflected pulse is inverted. This can be explained by Newton's third law, the support must exert an equal and opposite (downward) reaction force on the string. This downward force causes the pulse to invert upon reflection.



Now consider another case, where the pulse arrives at the end of a string that is free to move vertically. The tension at the free end is maintained by tying the string to a ring of negligible mass that to slide vertically on a smooth post. Again, the pulse will be reflected, but this time its displacement is not inverted. As the pulse reaches the post, it exerts a force on the free end, causing the ring to accelerate upward.

In the process, the ring "overshoots" the height of the incoming pulse and is then returned to its original position by the downward component of the tension.



Finally, we may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as shown in figure. When a pulse travelling on the light reaches the junction, part of it is reflected and inverted, and part of it is transmitted to the heavier string. As one would expect, the reflected pulse has smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary, when it is totally reflected.

When a pulse travelling on a heavy string strikes the boundary of a lighter string, as shown in figure, again part is reflected and part is transmitted. However, in this case, the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative densities of the two string.

Thus, the speed of a wave on a string increases as the density of the string decreases. That is ,pulse travels more slowly on a heavy string than a light string, if both are under the same tension. The following general rules apply to reflected waves:

When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be Inverted upon reflection.

An incident wave,

$$y_i(x, t) = a \sin (kx - \omega t)$$

(1) At a rigid boundary is reflected with a phase change of ϕ,

$$y_0(x, t) = -a \sin (kx + \omega t)$$

(2) At an open boundary is reflected without any phase change.

$$y_r(x, t) = a \sin (kx + \omega t)$$

Example 2.2: A bat emits ultrasonic sound of frequency 1000 kHz in air. If the sound meets a water surface, what is the wavelength of:

(A) the reflected sound,

(B) the transmitted sound?

[Speed of sound in air = 340 m/s and in water = 1486 m/s] [NCERT]

Ans. According to question,

$$v = 1000 \text{ kHz} = 10^6 \text{ Hz},$$
$$v_a = 340 \text{ m/s},$$
$$v_w = 1486 \text{ m/s}$$

(A) Wavelength of reflected sound,

$$\lambda_a = \frac{v_a}{v} = \frac{340}{10^6}$$
$$= 3.4 \times 10^{-4} \text{ m}$$

(B) Wavelength of transmitted sound,

$$\lambda_w = \frac{v_w}{v} = \frac{1486}{10^6}$$
$$= 1.486 \times 10^{-3} \text{ m}$$

Example 2.3: A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3.5 × 10^{-2} kg and its linear mass density is 4 × 10^{-2} kg/m. What is:

(A) the speed of the transverse wave on the string and

(B) the tension in the string? [NCERT]

Ans. Given,

$$v = 45 \text{ Hz},$$
$$M = 3.5 \times 10^{-2} \text{ kg},$$
$$m = 4 \times 10^{-2} \text{ kg/m}$$

$$l = \frac{M}{m} = \frac{3.5 \times 10^{-2}}{4 \times 10^{-2}}$$

$$= \frac{7}{8} \text{ m}$$

As it is vibrating in fundamental mode.

$$\lambda = \frac{2l}{n} = 1.75 \text{ m}$$

$$\lambda = \frac{2 \times \frac{7}{8}}{1} = 1.75 \text{ m}$$

(A) Speed. $v = \nu\lambda = 45 \times 1.75 = 78.75$ m/s

(B) $$v = \sqrt{\frac{T}{m}}$$

$$T = v^2 \times m$$
$$= (78.5)^2 \times 4 \times 10^{-2}$$
$$= 248.06 \text{ N}$$

TOPIC 3

STANDING WAVES AND NORMAL MODES

When two identical waves travelling in opposite directions along a straight line collide, a stationary wave is formed. A stationary wave will result from the superposition of the two waves. Only in a bounded medium can be a stationary wave form.

Analytical Method for Stationary Waves

From rigid end

$$y = -2a \sin kx \cos \omega t$$

This is the equation of stationary wave reflected from rigid end.

Velocity of particle.

$$v_p = \frac{dy}{dt} = 2a\omega \sin kx \sin \omega t$$

Strain. $$\frac{dy}{dx} = -2\,ak \cos kx \cos \omega t$$

Elasticity $$E = \frac{\text{Stress}}{\text{Strain}} = \frac{dp}{\frac{dy}{dx}}$$

Change in pressure.

$$dp = E\frac{dy}{dx}$$

From free end

$$y = 2\,a \sin \omega t \cos kx$$

This is the equation of stationary wave reflected from the free end.

Velocity of particle.

$$v_p = \frac{dy}{dt}$$

$$= 2a \cos \omega t \cos kx$$

Strain. $$\frac{dy}{dt} = -2ak \sin kx \sin \omega t$$

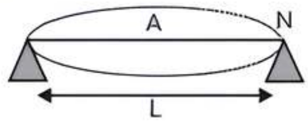
Change in pressure.

$$dp = E\frac{dy}{dx}$$

Stationary Transverse Waves

In this waves, ends are always nodes

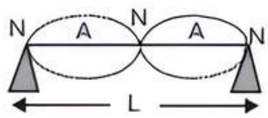
First harmonic.



$$\frac{\lambda}{2} = L$$

$$\Rightarrow \quad \lambda = 2L$$

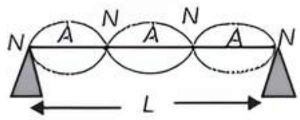
Second harmonic or first overtone.



$$\frac{2\lambda}{2} = L$$

$$\Rightarrow \quad \lambda = L$$

Third harmonic or second overtone.



$$\frac{3\lambda}{2} = L$$

$$\Rightarrow \quad \lambda = \frac{2L}{3}$$

For P^{th} harmonic $$\frac{P}{2} = L$$

$$\lambda = \frac{2L}{P}$$

Laws of Vibration of Stretched String

(1) Law of length: $n \propto \frac{1}{l}$ (T and m are constant).

(2) Law of tension: $n \propto \sqrt{T}$ (l and m are constant).

(3) Law of mass: $n \propto \frac{1}{\sqrt{m}}$ (l and T are constant).

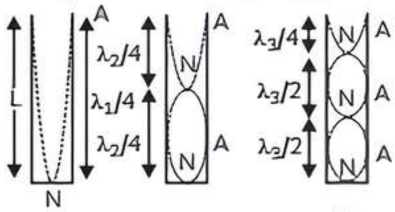
Stationary Longitudinal Waves

Standing waves are produced by superposition when two longitudinal waves of the same frequency and amplitude travel in opposite directions in a medium.

These waves are created by air columns in a uniformly sized cylindrical tube. Organ pipes are the tubes that produce sound.

Vibration of air column in closed organ pipe

If L is the length of pipe and λ be the wavelength and v be the velocity of sound in organ pipe then.



(a) (b) (c)

Fundamental frequency or first harmonic.

Case (a) $L = \frac{1}{4}.\lambda = 4L, n_1 = \frac{v}{\lambda} = \frac{v}{4L}$

First overtone or third harmonic.

Case (b) $L = \frac{3}{4}. \lambda = \frac{4L}{3}, n_2 = \frac{v}{\lambda} = \frac{3v}{4L}$

Second overtone or fifth harmonic.

Case (c) $L = \frac{5}{4}. \quad \lambda = \frac{4L}{5}, n_3 = \frac{v}{\lambda} = \frac{5v}{4L}$

When a closed organ pipe vibrates in m^{th} overtone then.

$$L = (2m + 1)\frac{v}{4}$$

so.

$$= \frac{4L}{(2m+1)}$$

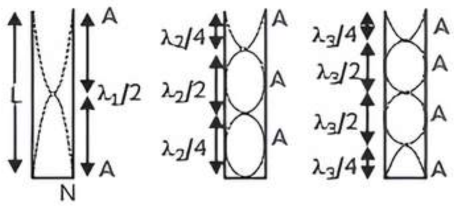
$$n = (2m + 1)\frac{v}{4L}$$

Hence. frequency of overtones is given by.

$$n_1 : n_2 : n_3 ...... = 1 : 3 : 5...$$

Vibration of air column in open organ pipe

If l is the length of the pipe and λ be the wavelength and v is the velocity of sound in organ pipe then.



(a) (b) (c)

Fundamental frequency or first harmonic.

Case (a) $L = \frac{1}{2}. \lambda = 2L, n_1 = \frac{v}{\lambda} = \frac{v}{2L}$

First overtone or third harmonic.

Case (b) $L = \frac{2}{2}.\lambda = \frac{2L}{2}, n_2 = \frac{v}{\lambda} = \frac{2v}{2L}$

Second overtone or fifth harmonic.

Case (c) $L = \frac{3}{2}.\lambda = \frac{2L}{3}, n_3 = \frac{v}{\lambda} = \frac{3v}{2L}$

Hence. frequency of overtones is given by the relation.

$$n_1 : n_2 : n_3 .... = 1 : 2 : 3 ....$$

When an open organ pipe vibrates in m^{th} overtone then.

$$L = (m + 1)\frac{v}{4}$$

so.

$$= \frac{4L}{m+1}.$$

$$n = (m + 1)\frac{v}{2L}$$

Example 2.4 **A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source? Will the same source be in resonance with the pipe, if both ends are open? Speed of sound = 340 m/s [NCERT]**

Ans. Given.

$$L = 20 \text{ cm} = 0.2 \text{ m}.$$

Natural frequency of the pipe

$$v_n = 430 \text{ Hz}.$$

$$v = 340 \text{ m/s}$$

$$v_n = (2n - 1)\frac{v}{4L} 430$$

$$= (2n - 1)\frac{340}{4 \times 0.2}$$

$$(2n - 1) = \frac{430 \times 4 \times 0.2}{340} = 1.02$$

$$2n = 2.02$$

and $n = 1.01$

It is 1^{st} normal mode of vibration.

In a pipe. with both ends open

$$v_n = n \times \frac{v}{2L}$$

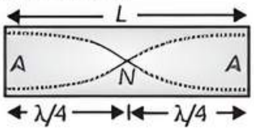
$$= \frac{n \times 340}{2 \times 0.2} = 430$$

$$n = \frac{430 \times 0.2 \times 2}{340} = 0.5$$

n should be an integer. therefore it is not in resonance with the source.

Example 2.5: **A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2.53 kHz. What is the speed of sound in steel?** **[NCERT]**

Ans. When the rod is clamped at the middle, a node is formed at the middle and 2 antipodes are formed at the two ends.



Here, $L = 100$ cm

$= 1$ m,

$\nu = 2.53$ kHz

$= 2.53 \times 10^3$ Hz

$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$$

$\lambda = 2L = 2$ m

$v = \nu\lambda = 2.53 \times 10^3 \times 2$

$= 5.06 \times 10^3$ m/s

Important

➥ *Only odd harmonics are produced in a closed organ pipe. Moreover, first overtone is the third harmonic, second overtone is the fifth harmonic and so on.*

Example 2.6: **A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.** **[NCERT]**

Ans. One end of the tube is open and the other end is closed by the piston, so it behaves as a closed organ pipe, which produces only odd harmonics.

The pipe is in resonance with the fundamental mode and the 3rd harmonic ($79.3 \approx 3 \times 25.5$)

For fundamental mode,

$$\frac{\lambda}{4} = l_1 = 25.5$$

$\lambda = 4 \times 25.5$

$\lambda = 102$ cm $= 1.02$ m

Speed, $v = 340 \times 1.02$

$= 346.8$ m/s

TOPIC 4

BEATS

Beats are the regular variations in sound intensity with time at a specific location caused by the superposition of two sound waves of slightly different frequencies.

For waves

$y_1 = a \sin \omega_1$

$t = a \sin 2\pi\nu_1 t$

And $y_2 = a \sin \omega_2$

$t = a \sin 2\pi \nu_2 t$

$Y = 2a \cos \pi (\nu_1 - \nu_2) t \sin \pi (\nu_1 + \nu_2)t$, is the required equation of beats.

Beat frequency,

$$-\nu_{Beat} = (\nu_1 - \nu_2)$$

Beat period, $$T = \frac{1}{\text{Beat frequency}}$$

$$= \frac{1}{\nu_1 - \nu_2}$$

Important

➥ *No. of beats formed per sec is known as beat frequency and time taken to complete one beat is known as beat time period.*

Phenomenon of beats can be used to determine the frequency of a tuning fork as follows:

If the known frequency of tuning fork A is ν_A and the unknown frequency of tuning fork B is ν_B. When both tuning forks A and B are sounded at the same time and produce beats of frequency ν.

$$\nu_B = \nu_A \pm \nu$$

The ± sign of ν is determined here by either loading wax or filing any of the tuning forks. If the beat frequency increases when the tuning fork B is loaded with wax and sounded with the tuning fork A, then $\nu_B = \nu_A - \nu$ If the frequency of the beats decreases, $\nu_B = \nu_A + \nu$. If the beat frequency increases when the tuning fork B is filled and sounded with the tuning fork A rather than loaded with wax, then $\nu_B = \nu_A + \nu$. If the beat frequency decreases, then $\nu_B = \nu_A - \nu$.

Example 2.7: **With a Sonometer wire length of 100 cm, a tuning fork produces 6 beats. If the length of the wire is reduced by 2 centimetres, the number of beats remains constant. What will be the fork's frequency?**

Ans. Let the frequency of tuning fork be n as the frequency of vibration

$$\text{frequency of string} \propto \frac{1}{\text{length of string}}$$

$$n \propto \frac{1}{l}$$

$$nl = \text{constant}$$

or $$n_1 l_1 = n_2 l_2$$

$$(n + 6) \times 98 = (n-6)\ 102$$

$$98\,n + 6 \times 98 = (102\,n) - (6 \times 102)$$

$$n = 300 \text{ Hz}$$

Example 2.8: When a column of air and a tuning fork are sounded together, they produce 4 beats per second. The lower note is provided by the tuning fork. The air temperature is 15°C. When the temperature drops to 10° degrees Celsius, the two produce 3 beats per second. Determine the frequency of the fork.

Ans. Let the frequency of the tuning fork be n Hz

Frequency of air column at 15°C = $n + 4$

Frequency of air column at 10°C = $n + 3$

From, $v = n\lambda$

we have,

$$v_{15} = (n + 4)$$

and $$v_{10} = (n + 3)$$

$$\frac{v_{15}}{v_{10}} = \frac{n+4}{n+3}$$

The speed of sound is directly proportional to the square root of the absolute temperature.

$$\frac{v_{15}}{v_{10}} = \sqrt{\frac{15+273}{10+273}}$$

$$\frac{n+4}{n+3} = \sqrt{\frac{288}{283}}$$

$$= \left(1 + \frac{5}{283}\right)^{\frac{1}{2}}$$

$$1 + \frac{1}{n+3} = 1 + \frac{1}{2} \times \frac{5}{283}$$

$$1 + \frac{1}{n+3} = 1 + \frac{5}{566}$$

$$\frac{1}{n+3} = \frac{5}{566}$$

$$n + 3 = 113$$

$$n = 110 \text{ Hz}$$

OBJECTIVE Type Questions

[1 mark]

Multiple Choice Questions

1. After 2 seconds, a man standing on a cliff claps his hand and hears the echo. The distance between the man and the reflection points is v_{sound} = 400 m/sec, if the sound is reflected from another mountain is:

(a) 340 m (b) 350 m

(c) 375 m (d) 400 m

Ans. *(d) 400 m*

Explanation: Given,

$$v_{sound} = 400 \text{ m/s}$$

Sound going to reflect back in 2 sec.

Sound reaches mountain in t = 1 sec.

Velocity of sound,

$$v_{sound} = \frac{d}{t}$$

$$d = v_{sound} \times t$$

$$t = 400 \times 1 = 400 \text{ m}$$

2. A tuning fork arrangement (pair) produces 7 beats per second with one fork at 350 cps. A small amount of wax is applied to the unknown fork, which produces 4 beats per second. The unknown fork's frequency is:

(a) 354 Hz (b) 360 Hz

(c) 370 Hz (d) 400 Hz

Ans. *(a) 354 Hz*

Explanation: Let the frequency of the unknown fork be ν.

As it produces, 7 beats with 350 Hz,

$$|\nu - 350| = 7$$

On placing wax, frequency will be,

$$\nu - 350 = \pm 4$$

$$\nu = 346 \text{ or } 354 \text{ Hz}$$

Waxing of fork decreases its frequency.

Since, beat frequency also drops, the frequency of unknown fork has to be, ν = 354 Hz.

Caution

➥ *Students must know that filing the prongs of a tuning fork raises the frequency and loading decreases the frequency.*

3. A 200 Hz wave travels along a string towards its fixed end. When this wave returns after reflection, a node is formed 15 cm away from the fixed end. The incident and reflected wave speeds are:

(a) 30 m/s (b) 60 m/s

(c) 90 m/s (d) 120 m/s

Ans. *(b) 60 m/s*

Explanation: Frequency (v) = 200 Hz and distance from fixed end = 15 cm = 0.15 m.

When a stationary wave is produced, the fixed end behaves as a node.

Thus, wavelength, $\lambda = 2 \times 0.15 = 0.3$ m.

Therefore, velocity

$$v = \nu\lambda$$
$$= 200 \times 0.3 = 60 \text{ m/s.}$$

Related Theory

- *Interference: Permanent Intensity pattern and it is the function of position [$I = f(x)$].*
- *Beats: Temporary intensity pattern and it is the function of time [$I = f(t)$].*

4. **A sonometer wire with a, M = 2 kg suspended mass is in resonance with a given tuning fork. The apparatus is transported to the moon, where the acceleration due to gravity is $\frac{1}{4}$ that of Earth. The value of M should be increased to achieve resonance on the moon is:**

(a) 5 kg **(b) 8 kg**
(c) 12 kg **(d) 25 kg**

Ans. *(b) 8 kg*

Explanation: $f \propto \sqrt{T} = \sqrt{mg}$

$$\therefore \quad m_1 g_1 = m_2 g_2$$
$$(2)g = m\left(\frac{g}{4}\right)$$
$$m = 8 \text{ kg}$$

5. **When vibrating with a tuning fork, a string under tension of 135 N produces 15 beats per second. The string vibrates in unison with the tuning fork when the tension is increased to 200 N. The tuning fork's frequency is then determined is:**

(a) 65 Hz **(b) 84.26 Hz**
(c) 90 Hz **(d) 112 Hz**

Ans. *(b) 84.26 Hz*

Explanation: Given,

$$T_1 = 135 \text{ N}$$
$$T_2 = 200 \text{ N}$$

and $b = 15$ beats/sec

As in first medium,

$$f_1 = \frac{1}{2l}\sqrt{\frac{T_1}{m}}$$
$$= f_0 - b \quad ...(i)$$

In the second condition,

$$f_2 = \frac{1}{2l}\sqrt{\frac{T_2}{m}}$$
$$= f_0 \quad ...(ii)$$

On dividing eqn. (i) by eqn. (ii).

We get, $$\sqrt{\frac{T_1}{T_2}} = \frac{f_0 - b}{f_0}$$
$$\sqrt{\frac{135}{200}} = \frac{f_0 - 15}{f_0}$$

Frequency of the tuning fork, $f_0 = 84.26$ Hz

6. **Two closed organ pipes have lengths of 0.80 m and 0.67 m. When they are sounded together, they produce 5 beats per second. The sound velocity will be:**

(a) 100.12 m/s **(b) 123.10 m/s**
(c) 125.625 m/s **(d) 132.64 m/s**

Ans. *(c) 125.625 m/s*

Explanation: For first closed organ pipe,

$$n_1 = \frac{v}{4 \times L_1} = \frac{v}{4 \times 0.80} = \frac{v}{3.2}$$

For second closed organ pipe,

$$n_2 = \frac{v}{4 \times L_2} = \frac{v}{4 \times 0.67}$$
$$n_1 - n_2 = 5$$
$$\frac{v}{3.2} - \frac{v}{4 \times 0.67} = 5$$
$$\frac{v}{3.2} - \frac{v}{2.68} = 5$$
$$v = 125.625 \text{ m/sec}$$

Caution

- *Students should know that for an organ pipe if f = constant.*

$$v \propto \lambda \text{ or } v \propto L \text{ and } f = \frac{v}{\lambda} = \text{constant}$$

i.e., the frequency of an organ pipe will remain unchanged if the ratio of speed of sound in to its wavelength remains constant.

7. **Sound waves of wavelength λ travelling in a medium with a speed of v m/s enter into another medium where its speed is $2v$ m/s wavelength of sound waves in the second medium is:**

(a) λ **(b) $\frac{\lambda}{2}$**
(c) 2λ **(d) 4λ**

[NCERT Exemplar]

Ans. *(c) 2λ*

Explanation: Wavelength of sound waves in the first medium,

$$\lambda = \frac{v}{\nu} \quad ...(i)$$

Wavelength of sound waves in the second medium,

$$\lambda' = \frac{2v}{v'} \quad \text{...(ii)}$$

As the frequency of the wave remains unchanged with the change in medium

$\therefore \quad v = v' \quad$ ...(iii)

Divide eqn. (ii) by eqn. (i), we get,

$$\frac{\lambda'}{\lambda} = 2$$

or $\lambda' = 2\lambda$

8. Equation of a progressive wave is given by $y = 0.2 \cos \pi\left\{(0.04t + 0.02x) - \frac{a}{6}\right\}$. The distance is expressed in cm and time in second. What will be the minimum distance between two particles having the phase difference of $\frac{\pi}{2}$?

(a) 4 cm (b) 8 cm

(c) 25 cm (d) 12.5 cm

[Delhi Gov. QB 2022]

Ans. *(c) 25 cm*

Explanation:

$$y = a\cos(\omega t + kx - \phi) \quad \text{...(i)}$$

$$y = 0.2\cos\left(0.04\pi t + 0.02\pi x - \frac{\pi^2}{6}\right) \quad \text{...(ii)}$$

Comparing eqn. (i) and (ii)

$\omega = 0.04\pi; k = 0.02\pi$

$$k = \frac{2\pi}{\lambda} = 0.02\pi$$

$$\lambda = \frac{2\pi}{0.02\pi} = 100 \text{ cm}$$

$$\Delta\phi = \frac{\pi}{2}$$

Hence, path difference between them

$$\Delta = \frac{\lambda}{2\pi} \times \Delta\phi$$

$$= \frac{\lambda}{2\pi} \times \frac{\pi}{2}$$

$$= \frac{\lambda}{4} = \frac{100}{4}$$

$$\Delta = 25 \text{ cm}$$

Assertion – Reason Questions

Two statements are given one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below:

(a) Both A and R are true and R is the correct explanation of A.

(b) Both A and R are true and R is not the correct explanation of A.

(c) A is true but R is false.

(d) A is false and R is also false.

9. Assertion (A): We cannot hear beats when two vibrating tuning forks with frequencies of 240 Hz and 300 Hz are held close together.

Reason (R): Because of the property of hearing persistence, beats cannot be clearly heard.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: The principle of superposition does not state that the frequencies oscillation should be nearly equal. And for beats to be heard the condition is that the difference in frequencies of the two oscillations be not more than 16 times per sec for a normal human ear to recognize it. Hence, we cannot hear beats in the case of two tuning forks vibrating at frequencies 256 Hz and 512 Hz respectively.

10. Assertion (A): Harmonics are the notes of frequencies which are integral multiple of the fundamental frequency.

Reason (R): Tones of frequencies higher than fundamental notes are called overtones.

Ans. *(b) Both A and R are true and R is not the correct explanation of A.*

Explanation: Integral multiple of something simply means a quantity multiplied by an integer. The overtones with frequencies which are integral multiples of the fundamental frequency are called harmonics. Hence, all harmonic are overtones. But overtones which are non-integrals multiples of the fundamental frequency are not harmonics. The waveforms of all sounds, apart from a basic sine wave, consist of the fundamental tone and many other tones of different frequencies. Non-fundamental tones that are whole-number multiples of the fundamental tone are known as overtones or harmonics.

11. Assertion (A): When a moth makes its way along the sand within a few tens of centimetres of a sand spider, the spider immediately turns and dashes towards the moth.

Reason (R): When a moth disturbs the sand, pulses are sent along its surface. The first set of pulses is longitudinal, while the second set is transverse.

Ans. *(a) Both A and R are true and R is the correct explanation of A.*

Explanation: When beetle moves along the sand it sends two sets of pulses, one longitudinal and the other transverse. Scorpions has the capacity to intercept the waves. By getting a sense of time interval between receipt of these two waves, it can determine the distance of moth also.

The sand spider uses waves of both transverse and longitudinal motion to locate its prey. When a moth even slightly disturbs the sand, it sends pulses along the sand's surface. One set of pulses is longitudinal, travelling with speed v_L. A second set is transverse travelling with speed v_T. The spider with its eight legs spread roughly in a circle intercepts the faster longitudinal pulses first and learns the direction of the moth, it is in direction of whichever leg is disturbed earliest by the pulses. The spider then senses the time interval (Δt) between the first interception and the interception of the slower transverse waves and uses it to determine the distance d to the moth.

$$\Delta t = \frac{d}{v_T} - \frac{d}{v_L}$$

which gives the scorpion a perfect fix on the moth.

12. **Assertion (A):** The basis of Laplace correction was that, exchange of heat between the region of compression and rarefaction in air is not possible.

Reason (R): Air is a bad conductor of heat and velocity of sound in air is large. [Delhi Gov. QB 2022]

Ans. (a) *Both A and R are true and R is the correct explanation of A.*

Explanation: According to the Laplace correction, the change in gas volume and pressure brought on by sound waves passing through it is an adiabatic process rather than an isothermal one. Since the air is a poor conductor, it prevents the transfer of heat between the layers and environment.

CASE BASED Questions (CBQs)

[4 & 5 marks]

Read the following passages and answer the questions that follow:

13. *A Sonometer is defined as the device that is used for demonstrating the relationship between the frequency of the sound that is produced by the string when it is plucked and the tension, length, and mass per unit length of the string. The sound is produced in the transverse standing wave in the string.*

(A) A Sonometer wire is vibrating in resonance with a tuning fork still be is resonance with the wire?

(B) Why do tuning forks have two prongs?

(C) Hospital uses an ultrasonic scanner to locate tumours in a tissue. What is the wavelength of sound in the tissue in which the speed of sound is 1.7 kms^{-1}? The operating frequency of the scanner is 4.2 MHz.

Ans. (A) When a wire of length L vibrates its resonant frequency in n^{th} mode after stretching it by a tension T, then frequency of n^{th} harmonic is, $v = \frac{n}{2L}\sqrt{\frac{T}{m}}$, here m is mass per unit length of stretched wire. Let in given two cases,

$$v_1 = \frac{n}{2L_1}\sqrt{\frac{T_1}{m_1}}$$

$$v_2 = \frac{n}{2L_2}\sqrt{\frac{T_2}{m_2}}$$

In given question,

$$T_1 = T_2 = T$$

$$m_1 = m_2 = m$$

as wire same $\quad L_2 = 2L_1$

$$\frac{v_1}{v_2} = \frac{n_1\sqrt{T}\sqrt{m} \times 2 \times 2L_1}{n_2\sqrt{T}\sqrt{m}\,2L_1} = \frac{2n_1}{n_2}$$

As the tuning fork is the same, i.e., in both harmonics n_1 and n_2 frequency of resonance same.

$\therefore \quad v_1 = v_2$

or $\quad \frac{2n_1}{n_2} = 1$

$$n_2 = 2n_1$$

(B) The two prongs of a tuning fork set each other is resonant vibrations and help to maintain the vibrations for a longer time.

(C) Speed of sound in the tissue.

$$v = 1.7 \text{ km/s}$$
$$= 1.7 \times 10^3 \text{ m/s}$$

Operating frequency of the scanner.

$$\nu = 4.2 \text{ MHz}$$
$$= 4.2 \times 10^6 \text{ Hz}$$

The wavelength of sound in the tissue is given as:

$$\lambda = \frac{v}{\nu}$$

$$\lambda = \frac{1.7 \times 10^3}{4.2 \times 10^5}$$

$$\lambda = 4.1 \times 10^{-4} \text{ m}$$

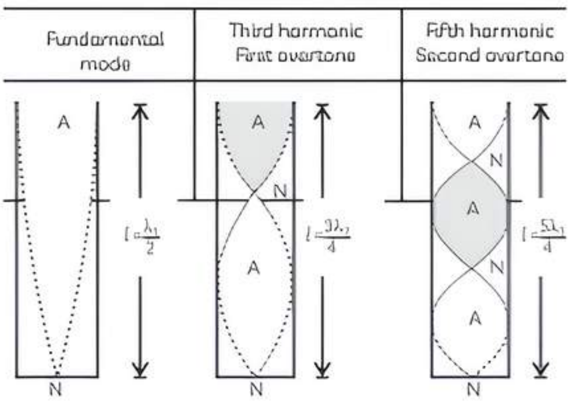
14. *Organ pipes are the musical instruments which are used for producing musical sound by blowing air into the pipe. Longitudinal stationary waves are formed on account of superimposition of incident and reflected longitudinal waves.*

Equation of standing wave.

$$y = 2a\cos\frac{2\pi t}{\lambda}\sin\frac{2\pi x}{\lambda}$$

Frequency or vibration, $n = \frac{v}{\lambda}$

Closed organ pipe



[Delhi Gov. QB 2022]

(A) **Assertion (A):** When we start filling an empty bucket with water, the pitch of sound produced goes on decreasing.

Reason (R): The frequency of man voice is usually higher than that of woman.

(a) Both A and R are true and R is the correct explanation of A.
(b) Both A and R are true and R is not correct explanation of A.
(c) A is true but R is false.
(d) A is false and R is also false.

(B) A closed pipe and an open pipe have their first overtones identical in frequency. Their lengths are in the ratio of:

(a) 1 : 2 (b) 2 : 3
(c) 3 : 4 (d) 4 : 5

(C) The first overtone in a closed pipe has a frequency:

(a) Same as the fundamental frequency of an open tube of same length.
(b) Twice the fundamental frequency of an open tube of same length.
(c) Same as that of the first overtone of an open tube of same length.
(d) None of the above.

(D) An empty vessel is partially filled with water, then the frequency of vibration of air column in the vessel:

(a) Remains the same
(b) Decreases
(c) Increases
(d) First increases, then decrease

(E) It is desired to increase the fundamental resonance frequency in a tube which is closed at one end. This can be achieved by:

(a) Replacing the air in the tube by hydrogen gas.
(b) Increasing the length of the tube.
(c) Decreasing the length of the tube.
(d) All of the above.

Ans. (A) *(d) A is false and R is also false.*

Explanation: A bucket can be treated as a pipe closed at one end. The frequency of the note produced $L = \frac{v}{4L}$, here L equal to depth of water level from the open end. As the bucket is filled with water L decreases, hence frequency increases. Therefore, frequency or pitch of sound produced goes on increasing. Also, the frequency of woman voice is usually higher than that of man.double of the closed pipe.

(B) *(c) 3 : 4*

It is given that

First overtone of closed pipe = First overtone of open pipe

$$\Rightarrow \quad 3\left(\frac{v}{4l_1}\right) = 2\left(\frac{v}{2l_2}\right)$$

where l_1 and l_2 are the lengths of closed and open organ pipes.

Hence, $$\frac{l_1}{l_2} = \frac{3}{4}$$

(C) *(d) None of the above.*

Explanation: First overtone for closes pipe

$$= \frac{3v}{4l}$$

Fundamental frequency for open pipe

$$= \frac{v}{2l}$$

First overtone for open pipe

$$= \frac{2v}{2l}.$$

(D) (c) *Increases*

Explanation: For closed pipe in general.

$$n = \frac{v}{4l}(2N-1)$$

$$\Rightarrow \qquad n \propto \frac{1}{l}$$

i.e., if length of air column decreases frequency increases.

(E) (d) *All of the above*

Explanation: Fundamental frequency for closed pipe

$$n = \frac{v}{4l}$$

where, $v = \sqrt{\frac{\gamma RT}{M}}$

$$\Rightarrow \qquad v \propto \frac{1}{\sqrt{M}}$$

$$\therefore \qquad M_{H_2} < M_{air}$$

$$\Rightarrow \qquad v_{H_2} < v_{air}$$

Hence, fundamental frequency with H will be more as compared to air.

Also $n \propto \frac{1}{l}$, hence if l decrease n increases

It is well known that $(n) = 2(n)$, hence all options are correct.

VERY SHORT ANSWER Type Questions VSA

[1 mark]

15. Assuming a simple harmonic disturbance in the water of a pond, how would you expect the amplitude of any given ripple to change with radius *R* as the ripple expands out?

Ans. The power of a wave of a given frequency and velocity of propagation is proportional to the square of the amplitude. Since falls off as $\frac{1}{R}$ for of our circular ripples, the amplitude of the wave decreases as $\frac{1}{\sqrt{R}}$.

16. A spherical sound wave emanates from a small whistle suspended from a ceiling of a very large room, emitting a single frequency harmonic wave. If the wave amplitude were 0.20 mm at R = 1.0 m, what is the amplitude at R = 3.0 m?

Ans. In this question, the power off as $\frac{1}{R^2}$, so the wave amplitude falls off as $\frac{1}{R}$. If the amplitude were 0.20 mm at R = 1.0 m, then it would be $\frac{1}{3}^{rd}$ of that amount, or 0.0667 mm, at R = 3.0 m.

17. At what temperature (in °C) will the speed of sound in air be 3 times of its speed at 0°C?

[NCERT Exemplar]

Ans.

$$v \propto \sqrt{T}$$

$$\frac{v_T}{v_0} = \sqrt{\frac{T}{T_0}}$$

$$v_T = 3v_0 \qquad \text{(Given)}$$

$$\frac{3v_0}{v_0} = \sqrt{\frac{T}{273+0}}$$

or $\sqrt{T} = 3\sqrt{273}$

$$T = 9 \times 273 = 2457 \text{ K}$$

or $T = 2457 - 273 = 2184°C$.

18. A train standing at the outer signal of a railway station blows a whistle of 400 Hz in still air. The train begins to move with a speed of 10 m/s towards platform. What is the frequency of sound for an observer standing on the platform?

(Sound velocity in air = 330 m/s).

[NCERT Exemplar]

Ans. Given, $v_0 = 400$ Hz, $v_s = 10$ m/s

Velocity of sound in air

$$v_a = 330 \text{ m/s}$$

Apparent frequency by an observer standing on platform,

$$v' = \frac{v_a}{(v_a - v_s)} v_0$$

$$= \frac{330 \times 400}{(330-10)} = \frac{825}{2}$$

$$= 412.5 \text{ Hz}$$

19. If two sound waves of frequencies 480 Hz and 536 Hz superpose, will they produce beats? Would you hear the beats? [Delhi Gov. QB 2022]

Ans. Yes, 56 beats per second will be produced by the sound waves. But because hearing is persistent, we would be unable to hear these beats.

SHORT ANSWER Type-I Questions (SA-I)

[2 marks]

20. A girl standing 4000 feet from a high cliff swings an axe at a tree stump and hears a faint echo 5.2 seconds later. What was the sound velocity in the air that day?

Ans. The sound created by the axe hitting the stump first travels the distance, x, to the cliff, where it is reflected and makes the return trip of the same distance to the man.

$$2x = v_p t,$$

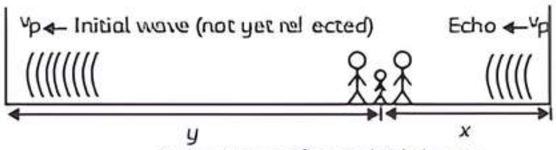
where t is the elapsed time for the echo and v_p is the velocity of sound in air. Substituting the given values for x and t in the equation,

$$v_p = \frac{2(4000\text{ ft})}{5.2\text{s}}$$

$$= 1538.46 \text{ ft/s}$$

21. A child standing with his parents somewhere between the two walls of a wide canyon shouts "hello". They hear two loud echoes, which one parent times with a stopwatch. The first echo arrived after an interval of 1.2 s, while the second arrived 1.8 s later. How wide is the canyon?

Ans.



A short time after a child shouts

As shown in figure, The family is clearly not midway between the two walls because the echoes took different times. Letting x and y be the respective distances to the near and far walls, we have,

$$2x = v_p t_1$$

$$= (1050 \text{ ft/s})(1.2\text{s})$$

$$= 1260 \text{ ft}$$

$$x = 630 \text{ ft}$$

22. A pipe 20 cm is closed at one end. Which harmonic mode of the pipe is resonantly excited by source 1237.5 Hz (velocity of sound in air = 330 m/s)? [NCERT Exemplar]

Ans. Given that

l = 20 cm = 0.2 m, n = 1237.5 Hz, v = 330 m/s

$$l = \frac{\lambda}{4}$$

Or $$\lambda = 4l$$

For fundamental frequency,

$$\nu_1 = \frac{v}{\lambda}$$

$$= \frac{v}{4l} = \frac{330}{4 \times 20 \times 10^{-2}}$$

$$= 412.5 \text{ Hz}$$

$$\nu \text{ (Given)} = 1237.5 \text{ Hz}$$

$$\frac{\nu(g)}{\nu_1} = \frac{1237.5}{412.5}$$

$$= \frac{3}{1}$$

23. Even after the breakup of one prong of tuning fork it produces a round of the same frequency, then what is the use of having a tuning fork with two prongs? [Delhi Gov. QB 2022]

Ans. A tuning fork's two prongs work together to create resonant vibrations that are sustained for a longer period of time.

SHORT ANSWER Type-II Questions (SA-II)

[3 marks]

24. A powerful firecracker is tossed in the air and explodes 5 m from a person walking nearby. The peak sound power generated by the explosion is 16 W.

(A) What is the intensity of sound that enters the person's ear?

(B) To how many decibels does this correspond?

(C) At what distance r from the explosion would the person's ear have to be if the sound was at the threshold of pain?

Ans. (A) Let the energy disperses a spherically symmetric shell away from the burst site. so. the intensity of sound is given by.

$$I = \frac{P}{4\pi r^2} = \frac{16}{4\pi \times (5)^2}$$

$$= 5.09 \times 10^{-2}\ W/m^2$$

(B) For converting this into decibels.

$$n = 10 \log \left[5.09 \times \frac{10^{-2}}{1.0} \times 10^{-12}\right]$$

$$= 107 dB$$

(C) Here, $n = 120$.

So. $$12 = \log\left(\frac{I}{I_0}\right).$$

$$I = I_0 \times 10^{12}$$
$$= 1.0\ W/m^2$$

Then $16\ W = (1.0\ W/m^2)(12.56)r^2$

or $r = 1.13$ m.

25. **A spherical sound wave emanates from a small whistle suspended from a ceiling of a very large room, emitting a single frequency simple harmonic wave.**

(A) If the power generated by the whistle is 0.0020 W, find the intensity of the spherical wave 1.0 m, 2.0 m, and 3.0 m from the source.

[Hint: Recall that the surface area of a sphere of radius r is $4\pi r^2$].

(B) Find the power passing through an imaginary circular window of area 12.0 cm^2, which is facing (parallel to) the wave fronts and at a distance of 3.0 m from the source.

Ans. (A) All the power must pass through any imaginary concentric spherical shell and by symmetry will flow out with equal intensity in all directions. For a spherical shell of radius r. the intensity would thus be:

$$I = \frac{P}{A} = \frac{P}{(4\pi r^2)}.$$

Substituting the value of P and the various r values into this relationship.

We get R = 1 m.
$I = 0.159\ mW/m^2$.
$r = 2$m.
$I = 0.0398\ mW/m^2$.
$r = 3$m.
$I = 0.0177\ mW/m^2$

(B) Since I represent area passing perpendicular to the imaginary window. we must have.

$$P_A = IA = (0.0177 \times 10^{-3}\ W)(12.0 \times 10^{-4}\ m^2)$$

$$= 0.212\ mW$$

26. **Show that when a string is fixed at its two ends vibrates in 1 loop, 2 loops, 3 loops and 4 loops, the frequencies are in the ratio 1 : 2 : 3 : 4.**

Ans. Let n be the number of loops in the string. The length of each loop is $\frac{\lambda}{2}$.

$$L = \frac{n\lambda}{2}.$$

$$\lambda = \frac{2L}{n}$$

$$v = \nu\lambda.$$

or $$\lambda = \frac{v}{\nu}$$

$$\frac{v}{\nu} = \frac{2L}{n}$$

$$\nu = \frac{n}{2L}v$$

$$\nu = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

For $n = 1$.

$$\nu_1 = \frac{1}{2L}\sqrt{\frac{T}{m}} = \nu_0$$

If $n = 2$, then

$$\nu_2 = \frac{2}{2L}\sqrt{\frac{T}{m}}$$
$$= 2\nu_o$$

If $n = 3$, then

$$\nu_3 = \frac{3}{2L}\sqrt{\frac{T}{m}}$$
$$= 3\nu_o$$

$$\nu_1 : \nu_2 : \nu_3 : \nu_4 = n_1 : n_2 : n_3 : n_4$$
$$= 1 : 2 : 3 : 4$$

27. **In reference to a wave motion, define the terms:**

(A) Amplitude

(B) Time period

(C) Frequency

(D) Angular frequency

(E) Wavelength and wave velocity.

[Delhi Gov. QB 2022]

Ans. (A) **Amplitude:** The highest displacement of a medium particle on either side of its mean location is referred to as the wave's amplitude. The meter is its S.I. unit (m).

(B) **Time period:** The length of time it takes for any string element to complete an oscillation is known as a wave's oscillation period.

(C) **Frequency:** The frequency of the waves is the number of vibrations created by a wave-producing particle of the medium in one second. It may also be thought of as the quantity of waves that travel through a spot in a second. Hertz is its SI unit (Hz).

(D) **Angular frequency:** The rate at which the waveform phase changes or the angular displacement of any wave element per unit of time is referred to as the angular frequency.

(E) **Wavelength:** The wavelength of a wave is the distance it travels when a medium particle vibrates in one unit of time. The meter is its S.I. unit (m)

Wave velocity: A wave's wave velocity is the distance it travels in one second. The S.I. unit is the meter per second (ms^{-1}).

LONG ANSWER Type Questions (LA)

[4 & 5 marks]

28. A cord of length, L = 3.0 m and total mass *m* = 400 g, is connected at one end to a vibrator, and at the other end to a very long and massive steel cable under tension *S* = 400 N. When the vibrator is turned on the cord is found to rapidly develop a large, stable, transverse standing wave consisting of five equal sections. The cable is observed to have a transverse travelling SHM wave to the right at 15 m/s.

(A) Find the wavelength and frequency of the standing wave of the cord.

(B) Find the wavelength and frequency of the travelling wave of the cable.

Ans. (A) Since the cord vibrates in five sections, and each section has a node at each end so that it is a half-wavelength.

We have, $\frac{\lambda}{2} = \frac{L}{5} = \frac{3}{5} = 0.60$ m.

$\lambda = 1.20$ m

The wavelength, the frequency can now be determined from a knowledge of the speed of propagation v_p, in the cord.

We have,

$$v_p = \left(\frac{S}{\mu}\right)^{\frac{1}{2}}$$

$$= \left(\frac{(400\text{ N})(3.0\text{ m})}{0.40}\right)^{\frac{1}{2}}$$

$$= 54.8 \text{ m/s}$$

$$f = \frac{v_p}{\lambda}$$

$$= \frac{54.8}{1.20} = 45.6 \text{ Hz}$$

(B) The frequency must be the same as the driving frequency of the cord which is $f = 45.6$ Hz.

The wavelength is now determined from the velocity of propagation in the steel cable.

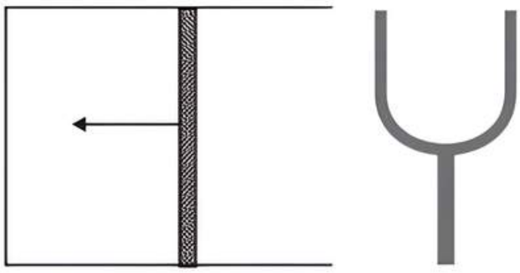
$$v_p = 15 \text{ m/s}$$

$$\lambda = \frac{v_p}{f}$$

$$= \frac{15}{45.6}$$

$$\lambda = 0.329 \text{ m}$$

29. A long tube has a close the way into the tube and a tuning fork is vibrating just in front of the tube, as shown figure. As the piston is slowly pulled (to the left in the figure) the sound gets very loud as it passes the 13 cm, 41 cm, and 69 cm marks as measured from the right end of the tube.

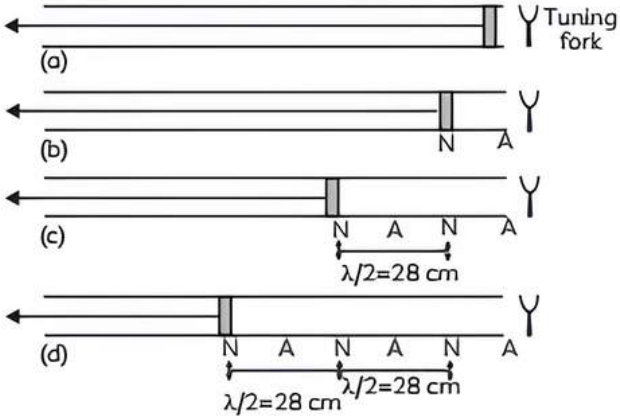


(A) What is the next point at which the loudness will rise?

(B) Assuming that the speed of sound is 350 m/s, find the frequency of the tuning fork.

Ans. (A) The tube acts like a closed organ pipe with regard to resonant standing waves. The tuning fork has a definite frequency, and an associated definite wavelength. As the piston is pulled back, first possible resonance occurs when a node is at the piston face and an anti-node is at, or just outside, the open end of the pipe-with the distance between node and anti-node corresponding to a quarter-wavelength back so that a second node occurs between the piston and the open end of the pipe.

Since the wavelength is fixed, one node remains at its original location, and the second node must be at the new position of the piston [Fig. (c)]. The distance between the nodes is precisely a half-wavelength and from the given data corresponds to:



$$\frac{\lambda}{2} = 41 \text{ cm} - 13 \text{ cm}$$
$$= 28 \text{ cm}$$
$$\lambda = 56 \text{ cm} \quad \text{...(i)}$$

The next resonance should occur when the piston is pulled sufficiently far that a third node occurs, the other two nodes maintaining at their prior locations (Fig. d) Again, we have the given data:

$$\frac{\lambda}{2} = 69 \text{ cm} - 41 \text{ cm}$$
$$= 28 \text{ cm}$$
$$\lambda = 56 \text{ cm} \quad \text{...(ii)}$$

This confirms our previous result and makes clear that the next resonance will occur when the piston is pulled another 28 cm further in, or at the 97 mark. Note that, the first data point of 13 is 1 cm short of a quarter wavelength, indicating that the anti-node is actually 1 cm outside the mouth of the pipe.

(B) From eqn. (i) and (ii).

We have, $\lambda = 56$ cm.

So, $$f = \frac{v_p}{\lambda} = \frac{350}{0.56}$$
$$= 625 \text{ Hz}$$

30. The earth has the radius 6400 km. The inner core of 1000 km radius is solid. Outside it, there is a region from 1000 km to a radius 3500 km which is in a molten state. Then again 3500 km to 6400 km, the earth is solid. Only longitudinal (P) waves can travel inside a liquid. Assume that P waves have a speed of 8 km/second in solid part and of 5 km/second in liquid part of the earth. An earthquake occurs at some place close to the surface of the earth. Calculate the time after which it will be recorded in a seismometer at a diametrically opposite point on the earth if wave travels along diameter.

[NCERT Exemplar]

Ans. Given that
$$r_1 = 1000 \text{ km}$$
$$r_2 = 3500 \text{ km}$$
$$r_3 = 6400 \text{ km}$$
$$d_1 = 1000 \text{ km}$$
$$d_2 = 3500 - 1000$$
$$= 2500 \text{ km}$$
$$d_3 = 6400 - 3500 \text{ km}$$

Solid distance diametrically,
$$= 2(d_1 + d_3)$$
$$= 2(1000 + 2900)$$
$$= 2 \times 3900 \text{ km}$$

Time taken by wave produced by earthquake in solid part,
$$= \frac{3900 \times 2}{8} \text{ sec}$$

Liquid part along diametrically
$$= 2d_2$$
$$= 2 \times 2500$$

Time taken by the seismic wave in liquid part $\frac{2 \times 2500}{5}$.

$$\text{Total time, } t = \frac{2 \times 3900}{8} + \frac{2 \times 2500}{5}$$
$$t = 2\left[\frac{3900}{8} + \frac{2500}{5}\right]$$
$$t = 2[487.5 + 500]$$
$$t = 2 \times 987.5$$
$$t = 1975 \text{ sec}$$
$$t = 32 \text{ min } 55 \text{ sec}$$

31. Show that the speed of sound in air increases by 61 cms^{-1} for every 1°C rise of temperature. [Delhi Gov. QB 2022]

Ans.
$$\frac{v_1}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273+0}}$$

where v_1, v_0 are the velocities of sound at T and T_0 respectively.

Therefore, $$\frac{v_1}{v_0} = \left(1 + \frac{t}{273}\right)^{\frac{1}{2}} = 1 + \frac{1}{2} \times \frac{1}{273},$$

neglecting the higher powers.

Therefore, $$v_1 = v_0\left(1 + \frac{t}{546}\right) = v_0 + v_0\left(\frac{t}{546}\right)$$

Therefore,
$$v_1 - v_0 = \frac{v_0 t}{546} = \frac{332 \times 1}{546}$$
$$= 0.608 = 61 \text{ cm/s}$$

Thus, the velocity of sound increases by 61 cm/s for every 1°C (or 1K) rise in the temperature.

NUMERICAL Type Questions

32. **Sound signals emitted by the two cars in a car race are detected by a detector on the straight track at the race's end point. The observed frequencies are 230 Hz and 260 Hz, while the original frequency of both cars is 200 Hz. The race concludes with the cars separated by 200 metres. Assume that both cars move at a constant speed and that the velocity of sound is 230 m/s. Determine the amount of time spent. (3m)**

Ans. Let the velocities of car 1 and car 2 be v_1 m/s and v_2 m/s.

Apparent frequencies of sound emitted by car 1 and car 2 as detected at end point are,

$$f_1 = f_0 \frac{v}{v - v_1}$$

and

$$f_2 = f_0 \frac{v}{v - v_2}$$

$$230 = 200 \frac{230}{220 - v_1}$$

and

$$260 = 200 \frac{260}{260 - v_2}$$

$$v_1 = 30 \text{m/s}$$

and $v_2 = 60$m/s

The distance between both the cars just when the 2nd car reach point B.

$$200\text{m} = v_2 t - v_1 t$$

$$t = 6.667 \text{ sec}$$

33. **When a 0.98 m long metallic wire is stressed, a 0.04 m extension is produced. When sounded with this stressed metallic wire, an organ pipe 1 m long and open at both ends produces 10 beats in its fundamental mode. The frequency of beats is reduced by reducing the stress in the wire. Determine the wire's Young's modulus. Metallic wire has a density of 10^4 kg/m^3 and a sound speed of 292 m/s in air. (3m)**

Ans. Frequency of the transverse vibration of the stretched string.

$$f_1 = \frac{1}{2(L + \Delta L)}\sqrt{\frac{T}{\mu}}$$

Here $Y = \frac{T}{A}\frac{L}{\Delta L}$

or $T = \frac{YA\Delta L}{L}$

and $\mu = A\rho$

So $f_1 = \frac{1}{2(L+\Delta L)}\sqrt{\frac{Y\Delta L}{L\rho}}$

When the stress in the wire is decreased, f_1 will decrease, consequently the beat frequency will decrease if $f_1 < f_{Pipe}$

$$f_1 - f_{Pipe} = 10$$

or $\frac{1}{2(L + \Delta L)}\sqrt{\frac{Y\Delta L}{L\rho}} - \frac{v}{2l} = 10$

Substituting the value, we get

$$\frac{1}{2(0.98 + 0.04)}\sqrt{\frac{Y \times 0.04}{0.98 \times 10^4}} - \frac{292}{2 \times 1} = 10$$

Simplifying, we get, $Y = 2.175 \times 10^{-6}$ N/m^2

34. **A set of 24 tuning forks is arranged so that each gives 4 beats per second with the previous one and the last sounds the octave of first. Find frequency of Ist and last tuning forks. [Delhi Gov. QB 2022](3m)**

Ans. It should be 4 instead of 44.

Octave of first means double the frequency of first.

Let the frequency of first fork $= n$

Then, frequency of last fork $= 2n$

No. of forks $= 24$

Number of beats between two adjacent forks $= 4$

Frequencies of forks will be:

$n, (n + 4), (n + 8), (n + 12), ... (n + 23 \times 4)$

$$n + 92 = 2n$$

$$n = 92 \text{ Hz}$$

Frequency of last fork,

$$= 2 \times 92$$

$$= 184 \text{ Hz}$$